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$\mathcal{N}=6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals

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Abstract: We construct three dimensional Chern-Simons-matter theories with gauge groups $\mathrm{U}(N)\times\mathrm{U}(N)$ and $\mathrm{SU}(N)\times\mathrm{SU}(N)$ which have explicit $\mathcal{N}=6$ superconformal symmetry. Using brane constructions we argue that the $\mathrm{U}(N)\times\mathrm{U}(N)$ theory at level k describes the low energy limit of N M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. At large N the theory is then dual to M-theory on $AdS_4\times S^7/\mathbf{Z}_k$. The theory also has a 't Hooft limit (of large N with a fixed ratio N/k) which is dual to type IIA string theory on $AdS_4\times\mathbf{CP}^3$. For $k=1$ the theory is conjectured to describe N M2-branes in flat space, although our construction realizes explicitly only six of the eight supersymmetries. We give some evidence for this conjecture, which is similar to the evidence for mirror symmetry in $d=3$ gauge theories. When the gauge group is $\mathrm{SU}(2)\times\mathrm{SU}(2)$ our theory has extra symmetries and becomes identical to the Bagger-Lambert theory.

Keywords: AdS-CFT Correspondence, Extended Supersymmetry, Chern-Simons Theories, 1/N Expansion.

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Contents

1. Introduction

In this paper we construct and study conformal field theories in three dimensions with $\mathcal{N} = 6$ supersymmetry, or a total of 12 real supercharges.

Highly supersymmetric three dimensional conformal field theories are interesting for various reasons. One is the construction of the theory describing the worldvolume of membranes in M-theory (M2-branes) at low energies. Conformal Chern-Simons theories were

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explored in [1] for this purpose, but these theories did not have enough supersymmetry. More recently a theory with $\mathcal{N} = 8$ supersymmetry was constructed by Bagger and Lambert [2] (see also [3]) and was conjectured to be related to a specific theory of M2-branes for $k = 1, 2$ [4, 5].

Another motivation to study three dimensional conformal field theories is that they could describe interesting conformal fixed points in condensed matter systems. From this point of view the highly supersymmetric versions are interesting toy models which are more solvable. Of course, Chern-Simons theories often arise in interesting condensed matter systems.

Finally, one is interested in examples of the AdS_4/CFT_3 correspondence as a way to study some portion of the landscape of four dimensional backgrounds of string theory with a negative cosmological constant (which admit AdS_4 solutions). Here we find an infinite sequence of theories with a discrete coupling k, the level of the Chern-Simons theory, such that for large k the field theory has a weak coupling description while for small k the theory becomes strongly coupled. For the highly supersymmetric theories that we consider, the size of the internal space is comparable to the radius of curvature of AdS_4, so these backgrounds do not really look four dimensional. However, this system might be a good stepping stone in order to study less symmetric and more realistic compactifications.

The main theory we consider is a $d = 3$, $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge theory with four complex scalar fields C_I $(I = 1, 2, 3, 4)$ in the $(\mathbf{N}, \bar{\mathbf{N}})$ representation and their corresponding complex conjugate fields in the $(\bar{\mathbf{N}}, \mathbf{N})$ representation. In addition, we have fermionic partners of these bosonic fields. The gauge fields are not dynamical, and have a Chern-Simons action with opposite integer levels for the two gauge groups, k and $-k$. The theory we build has $\mathcal{N} = 6$ supersymmetry, and it is weakly coupled in the large k limit $(k \gg N)$.

These theories can be obtained as the IR limit of a particular brane construction, similar to the ones considered in [6, 7]. The brane construction preserves only $\mathcal{N} = 3$ supersymmetry.[1] The corresponding field theory has a matter content similar to what we had above, except that we now have dynamical gauge fields and their superpartners, which are massive due to the supersymmetric Chern-Simons terms. When we go to low energies we can integrate them out and recover the conformal theory with only Chern-Simons terms. The low energy theory has an enhanced $\mathcal{N} = 6$ supersymmetry.

The brane construction can be lifted to M-theory where it corresponds to M2-branes probing a transverse toric HyperKähler manifold [8] which describes two Kaluza-Klein monopoles at an angle. This eight dimensional space has a singularity at a single point, which is of the form $\mathbf{R}^8/\mathbf{Z}_k$, or $\mathbf{C}^4/\mathbf{Z}_k$, where the $\mathbf{Z}_k$ acts by rotating the phases of all four complex coordinates. Thus, the conformal field theory that we are discussing is equivalent to the low-energy theory on N M2-branes at this $\mathbf{C}^4/\mathbf{Z}_k$ singularity.

Taking the large N limit we can construct the gravity dual to these theories, which simply corresponds to M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$. This description is weakly curved when $N \gg k^5$. For larger values of k a circle in the M-theory description becomes small, and the

[1] There is also another brane construction leading to the same IR theory, which preserves $\mathcal{N} = 4$ supersymmetry.

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more appropriate description is in terms of type IIA string theory on $AdS_4 \times \mathbf{CP}^3$. This string theory background also describes the 't Hooft limit of the theory, $N, k \to \infty$ with $\lambda = N/k$ fixed. The radius of curvature scales as $R^2 \sim \sqrt{\lambda}\alpha'$, so it is weakly curved when $k \ll N$. We explore various properties of these solutions.

From the field theory point of view one can also consider $\mathcal{N} = 6$ theories with an $\mathrm{SU}(N) \times \mathrm{SU}(N)$ (or $(\mathrm{SU}(N) \times \mathrm{SU}(N))/\mathbf{Z}_N$) gauge group and the same structure as our theories. In the particular case of $N = 2$ one finds some extra symmetries, due to the fact that the $\mathbf{2}$ and $\bar{\mathbf{2}}$ representations of SU(2) are equivalent. These extra symmetries imply that the corresponding Chern-Simons-matter theory has $\mathcal{N} = 8$ supersymmetry, and in fact it is precisely equivalent to the Bagger-Lambert theory [2] in the presentation given in [9]. In our formulation of this theory the 3-algebra structure introduced in [2, 3] does not play any role.

In the case of $k = 1$ our theories should reduce to the theory of N M2-branes in flat space, and for $k = 2$ they reduce to the theory of N M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$. In these cases the theory should have additional supersymmetries ($\mathcal{N} = 8$) that are not apparent in the Lagrangian, since they exist only for these specific values of the coupling. We have not found a direct proof of this statement from the field theory point of view, but it is clear from the brane construction or string theory arguments, and we provide various tests of this statement. A good analogy would be the theory of a compact boson in two dimensions, which is generically not supersymmetric but becomes supersymmetric at a special value of the radius of the circle [10]. In any case, we claim that the sequence of theories that we explore contains the $\mathcal{N} = 8$ M2-brane theory as a particular example (whose field theory description is strongly coupled).

This paper is organized as follows. In section two we construct the three dimensional theories we will be considering and analyze their properties. In section three we discuss the brane construction of the same theories, and argue that they reduce to M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. In section four we discuss aspects of the gravity dual of these theories. We end in section five with some conclusions and open problems. Two appendices provide some technical details.

2. Chern-Simons-matter theories with $\mathcal{N} \geq 6$ superconformal symmetry

In this section we construct and analyze Chern-Simons matter theories with gauge groups $\mathrm{U}(N) \times \mathrm{U}(N)$ and $\mathrm{SU}(N) \times \mathrm{SU}(N)$ which have an explicit $\mathcal{N} = 6$ superconformal symmetry. We begin in section 2.1 by reviewing the construction of Chern-Simons-matter theories with $\mathcal{N} = 2$ and $\mathcal{N} = 3$ supersymmetry, which exist for arbitrary gauge groups and matter content. In section 2.2 we show that for specific gauge groups and matter content, the supersymmetry is enhanced to $\mathcal{N} = 6$. In section 2.3 we compute the moduli space of the $\mathrm{U}(N) \times \mathrm{U}(N)$ theories at level k, and show that it is the same as that of N M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. In section 2.4 we compute the spectrum of chiral operators and Wilson lines in these theories. In section 2.5 we discuss the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ case. In section 2.6 we argue that for $N = 2$ these theories have an enhanced $\mathcal{N} = 8$ supersymmetry, and we compare with the Bagger-Lambert theories [2].

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2.1 A review of $\mathcal{N}=2,3$ Chern-Simons-matter theories

Chern-Simons theories in $2+1$ dimensions are simple examples of topological theories. When they are coupled to matter fields, the theory is no longer topological; however, in some cases it is still conformally invariant. A specific example which was argued [11, 12] to be exactly conformal is the case of $\mathcal{N}=2$ supersymmetric Chern-Simons-matter theories with no superpotential. The field content of such theories includes a vector multiplet V (which is the dimensional reduction of the four dimensional $\mathcal{N}=1$ vector multiplet) in the adjoint of the gauge group G, and chiral multiplets Φ_i in representations R_i of this group. The kinetic term for the chiral multiplets takes the usual form (which is the dimensional reduction of the four dimensional kinetic term). However, the kinetic term for the vector multiplet is replaced by a supersymmetric Chern-Simons term. The form of this term in superspace is somewhat awkward, but in components (in Wess-Zumino gauge) it takes the simple form[2]

$$S_{\rm CS}^{\mathcal{N}=2}=\frac{k}{4\pi}\int \mathrm{Tr}\left(A\wedge dA+\frac{2}{3}A^3-\bar{\chi}\chi+2D\sigma\right), \tag{2.1}$$

where χ is the gaugino, D is the auxiliary field of the vector multiplet, and σ is the real scalar field in the vector multiplet (coming from the A_3 component of the gauge field when we dimensionally reduce from $3+1$ dimensions). For non-Abelian theories the level k is quantized; for $\mathrm{SU}(N)$ or $\mathrm{U}(N)$ theories it is quantized to be an integer when the trace in (2.1) is in the fundamental representation.

Note that there is no kinetic term for any of the fields in the vector multiplet, so they are all auxiliary fields. The kinetic term for the chiral multiplets includes couplings $-\bar{\phi}_i\sigma^2\phi_i-\bar{\psi}_i\sigma\psi_i$, which are the dimensional reduction of the kinetic terms in the fourth direction. In addition we have the usual D term coupling $\bar{\phi}_iD\phi_i$. We can integrate out the D field and obtain $\sigma=-\frac{4\pi}{k}(\bar{\phi}_iT^a_{R_i}\phi_i)T^a$ (where T^a are the generators of the group in the fundamental representation normalized so that $\mathrm{Tr}(T^aT^b)=\frac{1}{2}\delta^{ab}$). Integrating out also χ, the action takes the form (in components) [11, 12]

$$\begin{aligned}S^{\mathcal{N}=2}=&\int\frac{k}{4\pi}\mathrm{Tr}\left(A\wedge dA+\frac{2}{3}A^3\right)+D_\mu\bar{\phi}_iD^\mu\phi_i+i\bar{\psi}_i\gamma^\mu D_\mu\psi_i\\&-\frac{16\pi^2}{k^2}(\bar{\phi}_iT^a_{R_i}\phi_i)(\bar{\phi}_jT^b_{R_j}\phi_j)(\bar{\phi}_kT^a_{R_k}T^b_{R_k}\phi_k)-\frac{4\pi}{k}(\bar{\phi}_iT^a_{R_i}\phi_i)(\bar{\psi}_jT^a_{R_j}\psi_j)\\&-\frac{8\pi}{k}(\bar{\psi}_iT^a_{R_i}\phi_i)(\bar{\phi}_jT^a_{R_j}\psi_j).\end{aligned} \tag{2.2}$$

Classically, this action includes only marginal couplings in $2+1$ dimensions, and it has been argued [12] that it cannot be renormalized beyond a possible one-loop shift of k (based on the integrality of k) so that it is conformally invariant also at the quantum level.

A simple generalization of the theories above has $\mathcal{N}=3$ supersymmetry [13, 14]. To obtain this we need to begin with the field content of an $\mathcal{N}=4$ supersymmetric theory, namely we need to add to the vector multiplet an additional auxiliary chiral multiplet φ in the adjoint representation, and we need to assume that the chiral multiplets come

[2] We will generally adhere to the notations and conventions of [12], except for some factors of 2 in the $\mathcal{N}=3$ superpotentials which seem to be essential for obtaining theories with $\mathcal{N}=3$ supersymmetry.

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in pairs $\Phi_i, \tilde{\Phi}_i$ in conjugate representations of the gauge group (together these form a hypermultiplet). The action then includes the usual $\mathcal{N}=4$ superpotential $W = \tilde{\Phi}_i \varphi \Phi_i$, and the Chern-Simons term (which breaks the $\mathcal{N}=4$ supersymmetry to $\mathcal{N}=3$) includes an additional superpotential $W = -\frac{k}{8\pi}\mathrm{Tr}(\varphi^2)$. Since there is no kinetic term for φ it can simply be integrated out, leading to a superpotential

$$W = \frac{4\pi}{k}(\tilde{\Phi}_i T^a_{R_i} \Phi_i)(\tilde{\Phi}_j T^a_{R_j} \Phi_j). \tag{2.3}$$

So, the action is the same as (2.2) above (including the same terms for the conjugate chiral multiplets), with the addition of this marginal superpotential, whose coefficient is determined by the $\mathcal{N}=3$ supersymmetry, so that it is also not renormalized.

Another way to obtain the $\mathcal{N}=3$ SCFTs described above is to start from the $\mathcal{N}=4$ supersymmetric Yang-Mills gauge theory with the same matter content, and to deform it by adding a Chern-Simons term for the vector multiplet. Using a standard normalization for the kinetic terms of the vector multiplet with a $1/g^2_{\mathrm{YM}}$ in front, all components of the $\mathcal{N}=4$ vector multiplet then have mass $m = g^2_{\mathrm{YM}} k/4\pi$. At low energies, compared to this mass scale, these fields may be integrated out, leading to the effective action described above. In the adjoint supermultiplets we have three massive fermions with spin $+1/2$ and one with spin $-1/2$. When we integrate them out we shift the Chern-Simons level by $k \to k - N$, but this cancels with the shift that comes from the contribution of the gauge field. In the end the Chern-Simons level is not changed, see [14] for a more detailed discussion.

Theories with $\mathcal{N}$ superconformal symmetries in $d = 2{+}1$ dimensions have an $\mathrm{SO}(\mathcal{N})$ R-symmetry, which in our case is $\mathrm{SO}(3)$, or more precisely $\mathrm{SU}(2)_R$. In the vector multiplet, the fermions are a triplet and a singlet of $\mathrm{SU}(2)_R$, and the three scalar fields (σ and two from φ) are a triplet (as are the three auxiliary fields). In the chiral multiplets, all fields are doublets of $\mathrm{SU}(2)_R$. For instance, the lowest component of Φ_i, together with the complex conjugate of the lowest component of $\tilde{\Phi}_i$ (which is in the same representation of the gauge group as Φ_i) form a doublet of this symmetry. In addition, we have a $\mathrm{U}(N_f)$ flavor symmetry whenever we have N_f matter fields in the same representation of the gauge group.

As we reviewed, Chern-Simons theories with $\mathcal{N}=3$ supersymmetry can be written rather generally. Theories with $\mathcal{N}=4$ supersymmetry were recently constructed in [15] , see also [16].[3]

2.2 A special case with $\mathcal{N}=6$ supersymmetry

In this paper we will focus on a special case of the $\mathcal{N}=3$ construction above, with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ (or $\mathrm{SU}(N) \times \mathrm{SU}(N)$), and with two hypermultiplets in the bifundamental representation; we will denote the bifundamental chiral superfields by A_1, A_2 and the anti-bifundamental chiral superfields by B_1, B_2. We use a notation similar to the one in the Klebanov-Witten theory [18] since our field content is the same (albeit in one lower dimension), and some of the interactions will also be similar to that theory. We will also choose the Chern-Simons levels of the two gauge groups to be equal but opposite in sign.

[3]Theories with $\mathcal{N}=4$ but a modified superalgebra were written in [17].

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Before integrating out the vector multiplet, the superpotential in this theory takes the form

$$W=\frac{k}{8\pi}\mathrm{Tr}(\varphi_{(2)}^2-\varphi_{(1)}^2)+\mathrm{Tr}(B_i\varphi_{(1)}A_i)+\mathrm{Tr}(A_i\varphi_{(2)}B_i). \tag{2.4}$$

Integrating out the auxiliary fields $\varphi_{(i)}$ (as in [18], when we think about that theory as arising by flowing from a four dimensional $\mathcal{N}=2$ gauge theory), we get a superpotential (which is just (2.3) for this special case)

$$W=\frac{2\pi}{k}\mathrm{Tr}(A_iB_iA_jB_j-B_iA_iB_jA_j)=\frac{4\pi}{k}\mathrm{Tr}(A_1B_1A_2B_2-A_1B_2A_2B_1). \tag{2.5}$$

As described above, we can obtain this theory by starting from the $2+1$ dimensional $\mathcal{N}=4$ gauge theory with the same field content, deforming it to an $\mathcal{N}=3$ theory by the $\mathcal{N}=3$ supersymmetric Chern-Simons term, and flowing to low energies compared to the mass scale $m=g_{\mathrm{YM}}^2k/4\pi$.

Naively, the theory discussed above has only an SU(2) flavor symmetry rotating the A's and B's together (we will discuss additional U(1) flavor symmetries below). However, the superpotential (2.5) actually has a bigger flavor symmetry, since it can be written as

$$W=\frac{2\pi}{k}\epsilon^{ab}\epsilon^{\dot{a}\dot{b}}\mathrm{Tr}(A_aB_{\dot{a}}A_bB_{\dot{b}}), \tag{2.6}$$

which exhibits explicitly an $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry acting separately on the A's and on the B's (as in [18]). All other terms in our action (2.2) obviously have this bigger symmetry, so we conclude that the full $\mathcal{N}=3$ Chern-Simons-matter theory in this special case has this enhanced global symmetry.

However, as we discussed above, these theories also have an $\mathrm{SU}(2)_R$ symmetry under which the bosonic fields A_1 and B_1^* transform as a doublet (as do A_2 and B_2^*).[4] Obviously, this symmetry does not commute with the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry of the previous paragraph. The two symmetry transformations together generate an SU(4) symmetry, under which the four bosonic fields $C_I\equiv(A_1,A_2,B_1^*,B_2^*)$ transform in the $\mathbf{4}$ representation. Since $\mathrm{SU}(2)_R$ is an R-symmetry, we see that the supercharges cannot be singlets under this SU(4). Since general $d=3$ SCFTs have an $\mathrm{SO}(\mathcal{N})$ R-symmetry (with the supercharges in the fundamental representation), we see that we must have at least $\mathcal{N}=6$ supersymmetry. In the way that we wrote down the action, only the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ subgroup of the $\mathrm{SO}(6)_R$ symmetry is manifest; note that the $\mathrm{SU}(2)_R$ transformation mixes the scalar potential coming from the superpotential W with the scalar potential terms in (2.2). But the full theory has an $\mathcal{N}=6$ superconformal symmetry, which could be explicitly written down by performing $\mathrm{SU}(2)\times\mathrm{SU}(2)$ transformations on the generators of the original $\mathcal{N}=3$ superconformal symmetry. In appendix A.1 we explicitly verify that the full scalar potential is indeed invariant under $\mathrm{SO}(6)_R$.

The full global symmetry of the theories we wrote down is $\mathrm{SO}(6)_R\times\mathrm{U}(1)_b$. In the $\mathrm{SU}(N)\times\mathrm{SU}(N)$ case the $\mathrm{U}(1)_b$ symmetry is just the usual baryon number symmetry,

[4] In the previous paragraph A_i and B_i were denoting chiral superfields. In this paragraph they denote bosonic fields which are the $\theta=\bar{\theta}=0$ components of the chiral superfields. Hopefully this will not cause confusion.

under which the A_i have charge $(+1)$ and the B_i have charge (-1). In the $\mathrm{U}(N) \times \mathrm{U}(N)$ theory the baryon number symmetry is gauged by a gauge field A_b, but there is also an extra gauged $\mathrm{U}(1)_{\tilde{b}}$ symmetry which only couples through an $kA_b \wedge F_{\tilde{b}}$ type coupling. Thus, this theory also has a $\mathrm{U}(1)$ global symmetry which is generated by the current $J = \frac{k}{4\pi} * F_{\tilde{b}}$, and the equation of motion of A_b sets this current to be exactly equal to the $\mathrm{U}(1)_b$ current described above. (Naively there is also a global symmetry coming from $*F_b$, but the equation of motion of $A_{\tilde{b}}$ implies that this acts trivially.) Note that in this case the flux quantization condition on $F_{\tilde{b}}$ implies that the corresponding charges are all integer multiples of k. So we should really say that in this case the $\mathrm{U}(1)$ current is $\tilde{J} = J/k$.

The coupling constant of the field theories we wrote is $1/k$, so that for large k they are weakly coupled. However, in the large N limit with N/k fixed, as in the 't Hooft limit of standard gauge theories with adjoints and bifundamentals [19], there is an expansion in $1/N^2$ (whose leading order is given by planar diagrams), and the effective coupling constant in the planar diagrams is the 't Hooft coupling $\lambda \equiv N/k$. Thus, the theories we discuss are weakly coupled for $k \gg N$, and are strongly coupled when $k \ll N$. We will see below that in the strongly coupled regime, these theories have an alternative (dual) description that is weakly coupled (when $N \gg 1$). In the 't Hooft limit the dual is a weakly coupled string theory.

Finally, let us note that the theory is invariant under a parity symmetry which also exchanges the two gauge groups, and acts as charge conjugation on the fields, $C^I \to (C^I)^\dagger$ (and similarly for the fermions).

2.3 The moduli space of the $\mathrm{U}(N) \times \mathrm{U}(N)$ theories

We suspect that with this amount of supersymmetry the classical moduli space does not receive any quantum corrections. We begin by analyzing the moduli space in the case $N = 1$. In this case the superpotential (2.5) vanishes, and so do the last two lines in (2.2). (We have in this case $\sigma_{(1)} = \sigma_{(2)} = (2\pi/k)(|A_i|^2 - |B_i|^2)$, and all the couplings involve $\sigma_{(1)} - \sigma_{(2)}$.) Thus, this theory is simply a free theory of 4 superfields C_I (two of which are chiral superfields and two are anti-chiral superfields), with two gauge transformations acting as $A_{(1)} \to A_{(1)} - d\Lambda_{(1)}$, $A_{(2)} \to A_{(2)} - d\Lambda_{(2)}$, $C_I \to e^{i(\Lambda_{(1)} - \Lambda_{(2)})} C_I$, and with a Chern-Simons term

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int (A_{(1)} \wedge dA_{(1)} - A_{(2)} \wedge dA_{(2)}). \tag{2.7}$$

We can analyze the effect of the gauge transformation on the moduli space in two different ways. First, we can gauge-fix the gauge fields to zero. Naively we then obtain a moduli space which is just $\mathbf{C}^4$. However, this gauge-fixing leaves gauge transformations in which the Λ's are constant everywhere unfixed. Generally, in the presence of a boundary, such gauge transformations are not symmetries of the action, since

$$\delta S_{\mathrm{CS}} = \frac{k}{2\pi} \int_{\mathrm{boundary}} (\Lambda_{(1)} \wedge F_{(1)} - \Lambda_{(2)} \wedge F_{(2)}). \tag{2.8}$$

However, the gauge field strengths are quantized, $\int F_{(i)} \in 2\pi\mathbf{Z}$ for an integral over any closed 2-manifold. Thus, for the action to transform by 2π times an integer in a

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general configuration, we must have $\Lambda_{(i)} = 2\pi n/k$ for some integer n. Dividing by these additional identifications, we obtain that the moduli space is actually $\mathbf{C}^4/\mathbf{Z}_k$, where the $\mathbf{Z}_k$ symmetry acts as $C_I \to e^{2\pi i/k} C_I$. Thus, in this case our conformal theory is simply the supersymmetric sigma model on this orbifold, which has a manifest SU(4) symmetry.

Equivalently, we can derive the same result as follows (as done in a similar context in [4]). The Abelian Chern-Simons term can be written as $S_{\mathrm{CS}} = \frac{k}{4\pi}\int A_b \wedge F_{\tilde{b}}$, where $F_{\tilde{b}} = dA_{\tilde{b}}$, $A_b = A_{(1)} - A_{(2)}$, and $A_{\tilde{b}} = A_{(1)} + A_{(2)}$. The field $A_{\tilde{b}}$ only appears in the action in this Chern-Simons term, so we can dualize it into a scalar by treating $F_{\tilde{b}}$ rather than $A_{\tilde{b}}$ as the basic variable, and adding a Lagrange multiplier τ with

$$S_\tau = \frac{1}{4\pi}\int \tau(x)\epsilon^{\mu\nu\lambda}\partial_\mu F_{\tilde{b}\nu\lambda}. \tag{2.9}$$

The quantization condition on $F_{\tilde{b}}$ (which is the same as above) implies that τ is periodic with a period 2π. The equation of motion of $F_{\tilde{b}\mu\nu}$ implies that $A_{b\mu} = (1/k)\partial_\mu\tau$, and the kinetic terms of the scalar fields in C_I become $|D_\mu C_I|^2 = |\partial_\mu C_I + \frac{i}{k}C_I\partial_\mu\tau|^2$. Gauge invariance now implies that the remaining gauge transformation $C_I \to e^{i\theta(x)}C_I$ takes $\tau \to \tau - k\theta(x)$. If we now gauge-fix this remaining transformation by taking $\tau = 0$, we see that we can still perform gauge transformations with $\theta = 2\pi/k$. Thus, again we obtain a sigma model on $\mathbf{C}^4/\mathbf{Z}_k$.

The generalization to the $\mathrm{U}(N)\times\mathrm{U}(N)$ case is straightforward. Whenever the $N \times N$ matrices C_I are all diagonal, it is easy to see that the scalar potential vanishes exactly, just like in the previous case. And, it is easy to check that for generic diagonal elements of these matrices, all off-diagonal elements are massive, so these diagonal configurations are the full moduli space of the theory. Requiring that all the matrices are diagonal breaks the gauge symmetry to $\mathrm{U}(1)^N \times \mathrm{U}(1)^N \times S_N$, where the S_N permutes the diagonal elements of all the matrices (for generic eigenvalues only a $\mathrm{U}(1)^N$ subgroup of this which does not act on the eigenvalues remains unbroken). Up to the permutation symmetry we simply obtain N copies of the $\mathrm{U}(1)\times\mathrm{U}(1)$ theory, with the same flux quantization conditions as before for each $\mathrm{U}(1)$ factor in the low-energy theory (note that the i'th element of the fundamental representation of $\mathrm{U}(N)$ carries charge one under the i'th $\mathrm{U}(1)$ and no charges under any other $\mathrm{U}(1)$'s). Thus, the moduli space in this case is simply $(\mathbf{C}^4/\mathbf{Z}_k)^N/S_N$.

Note that this is the same as the moduli space of N M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity in M-theory. This theory also has $\mathcal{N} = 6$ supersymmetry since the orbifold preserves an $\mathrm{SU}(4)\times\mathrm{U}(1)$ isometry symmetry out of the full SO(8), and the $\mathbf{8}_c$ representation of SO(8) which the spinors live in decomposes into $\mathrm{SU}(4)\times\mathrm{U}(1)$ representations as $\mathbf{6}_0 + \mathbf{1}_2 + \mathbf{1}_{-2}$ [20, 21].[5] For $k > 2$ the last two supercharges are projected out when we divide by $\mathbf{Z}_k$ in $\mathrm{U}(1)$, so we remain precisely with $\mathcal{N} = 6$ supersymmetry, while for $k = 1, 2$ the M2-brane theory has a larger $\mathcal{N} = 8$ supersymmetry. We conjecture that our Chern-Simons-matter theory is exactly the same as the theory of M2-branes on the orbifold. In particular for $k = 1$ it describes M2-branes in flat space, and for $k = 2$ it de-

[5] This is correct for one sign of the M2-brane charge. For the other sign of the M2-brane charge the spinor decomposes as $\mathbf{4}_1 + \mathbf{4}_{-1}$, so there is no supercharge that is a singlet under the $\mathrm{U}(1)$ and we do not preserve any supersymmetry if $k > 1$ [20].

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scribes M2-branes probing an $\mathbf{R}^8/\mathbf{Z}_2$ singularity; in these cases there is an enhanced $\mathcal{N}=8$ supersymmetry. We have shown that the moduli spaces of these theories are identical, and we will provide further evidence for this conjecture below.

For $N=1$ the conjecture was proven above. In particular, for $k=1$ and $k=2$ we obtain the supersymmetric sigma models on $\mathbf{C}^4$ and on $\mathbf{C}^4/\mathbf{Z}_2$, respectively. These sigma models, which also arise as the low-energy theory on a single M2-brane in flat space or at an $\mathbf{R}^8/\mathbf{Z}_2$ singularity, have a larger $\mathcal{N}=8$ superconformal symmetry (and a corresponding SO(8) R-symmetry) which is not directly visible in the $\mathrm{U}(1)\times\mathrm{U}(1)$ action that we wrote down for these theories.

Another interesting point is that if we give a vacuum expectation value to one of the fields of the form $C_I=(\Lambda k)^{1/2}I_{N\times N}$, then the theory around this vacuum has an unbroken $\mathrm{U}(N)$ gauge symmetry. At energy scales of order Λ we transition from the conformal regime to the moduli space approximation. One can show, as in [22], that the $\mathrm{U}(N)$ gauge fields actually become dynamical with $g^2_{\mathrm{YM}}\sim\Lambda/k$, and that at energy scales below Λ the theory reduces to the maximally ($\mathcal{N}=8$) supersymmetric $\mathrm{U}(N)$ gauge theory. This description is weakly coupled for a range of energies when $g^2_{\mathrm{YM}}N\ll\Lambda$, or $k\gg N$. This is also clear from the picture of the branes probing the $\mathbf{C^4}/\mathbf{Z_k}$ singularity. If the M2-branes are sitting away from the origin, then for large k the $\mathbf{Z}_k$ identification looks like an identification on a small circle transverse to the M2-branes, and there is an energy range where the theory reduces to the $\mathcal{N}=8$ super Yang-Mills theory of D2-branes. In terms of the brane probes the configuration is similar to the "deconstruction" configuration in [23]. An important difference is that here we do not increase the number of dimensions where the theory is defined.

2.4 Chiral operators and Wilson lines

From the point of view of $\mathcal{N}=2$ supersymmetry, we can obtain chiral primary operators by taking any gauge-invariant products of A's and B's, modulo the F-term equations. In particular, we can take operators of the form $\mathrm{Tr}((A_aB_{\dot a})^l)$ for $l=1,2,\ldots,$ in which the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ indices are multiplied symmetrically (since anti-symmetric combinations vanish in the chiral ring due to the F-term equations). In the $\mathcal{N}=6$ language, the A's are part of a $\mathbf{4}$ of $\mathrm{SU}(4)_R$, and the B's are part of a $\overline{\mathbf{4}}$, so these operators are a subset of operators in the l'th symmetric product of $\mathbf{4}$'s times the l'th symmetric product of $\overline{\mathbf{4}}$'s, with no contractions between $\mathbf{4}$'s and $\overline{\mathbf{4}}$'s. More precisely, the Dynkin labels of the resulting SU(4) representation are $(l,0,l)$. This full class of operators, which we can write schematically as $\mathrm{Tr}((C_IC_J^\dagger)^l)$, must be chiral in the full $\mathcal{N}=6$ theory. The scaling dimension of these operators is $\Delta=l$. None of these operators carry any charge under $\mathrm{U}(1)_b$.

How can we obtain any operators carrying $\mathrm{U}(1)_b$ charge ? Recall that in the $\mathrm{U}(N)\times\mathrm{U}(N)$ theories this charge is generated by $J=(k/4\pi)*F_{\tilde b}$, so the corresponding states must involve a non-zero magnetic flux in the diagonal $\mathrm{U}(1)$ gauge group. This is a flux on the S^2 surrounding the point where the operator is inserted. These operators are called 't Hooft operators or monopole operators and were discussed in detail in [24, 25]. Let us begin by discussing this in the $\mathrm{U}(1)\times\mathrm{U}(1)$ case. A configuration with n units of flux in each of the $\mathrm{U}(1)$'s has nk units of charge under J_b, and the equation of motion of A_b tells us that this configuration must also carry $(-nk)$ units of the "original" baryon number

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charge (given by the number of A's minus the number of B's). In the $\mathrm{U}(1) \times \mathrm{U}(1)$ theory, we can thus obtain additional gauge-invariant operators by taking a product of nk B_i's, and adding n units of flux in each $\mathrm{U}(1)$. It is also useful to understand how these states arise when we consider the theory on $S^2 \times \mathbf{R}$. In this case we can construct a well defined state as follows. The B fields are now massive (due to their conformal coupling to the curvature) and their lowest modes are created by harmonic oscillator creation operators. We can consider a state that has q oscillators excited, which would carry $-q$ units of the original baryon charge. These states carry charges $(-q, q)$ under the $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge fields. We can now also put in n units of flux for each $\mathrm{U}(1)$ gauge field on the S^2. Due to the Chern Simons action this flux configuration has baryon number charge nk. Thus we see that if we set $q = nk$ we get an invariant configuration. More explicitly, the equations of motion for the gauge fields have the form

$$\frac{k}{2\pi} F_{(1)} - *\frac{i}{2}(\bar{B} dB - B d\bar{B}) = 0\,, \qquad -\frac{k}{2\pi} F_{(2)} + *\frac{i}{2}(\bar{B} dB - B d\bar{B}) = 0. \tag{2.10}$$

Integrating this on S^2 in a configuration where $\int_{S^2} F_{(1)} = \int_{S^2} F_{(2)} = 2\pi n$ and where the total charge carried by the field B is $-q$ we see that both equations in (2.10) reduce to $kn = q$. A third way to think about this problem is the following. The operator B^{nk} is charged under the $\mathrm{U}(1)_b$ gauge symmetry. So we can attach a Wilson line that ends on it and which couples to the $\mathrm{U}(1)_b$ gauge field. This is a Wilson line with charges $(nk, -nk)$ under $\mathrm{U}(1) \times \mathrm{U}(1)$. Thus, we can think of an operator of the form $(e^{ik \int_\infty^0 A})^n B^{nk}(0)$. This operator appears to be non-local. However, due to the Chern-Simons terms, such a Wilson line is not observable since it is equivalent to a large gauge transformation with gauge parameters $(\epsilon_{(1)}, \epsilon_{(2)}) = (n\varphi, n\varphi)$ where φ is the angle around the Wilson line, see [26]. Then we conclude that we have a local operator. An equivalent description is in terms of n 't Hooft disorder operators [27] inserted at the position where we insert the field B^{nk}. These are operators which insert a flux of magnetic field on a sphere surrounding the point where the operator is inserted. In the language we used in our discussion around (2.9), we can write these operators as $e^{in\tau(x)}$. In this language the full operator is $e^{in\tau} B^{nk}$.

Finally, we conclude that the spectrum of chiral operators in this theory includes, in addition to the operators described in the first paragraph, operators formed from nk A_i's (or nk B_i's), plus fluxes or unobservable Wilson lines ending on them. Generalizing this using the $\mathrm{SU}(4)_R$ symmetry we deduce the existence of gauge-invariant operators of the form C^{nk} (or $(C^\dagger)^{nk}$) which are in the (nk)'th symmetric product of $\mathbf{4}$'s ($\overline{\mathbf{4}}$'s) and carry nk units ($-nk$ units) of $\mathrm{U}(1)_b$ charge.

In the $\mathrm{U}(N) \times \mathrm{U}(N)$ case we can perform a similar construction [27–29]. The end result is that operators of the form C^{nk} are allowed. Naively such operators would not be gauge-invariant. They would transform in the symmetric product of nk fundamentals of the first $\mathrm{U}(N)$ and nk anti-fundamentals of the second $\mathrm{U}(N)$. Let us first consider the operator C^k. The non-Abelian theory also contains 't Hooft operators, or monopole operators [28–30]. We can consider such operators with one unit of flux on each of the two $\mathrm{U}(N)$ gauge groups. In the presence of the Chern-Simons term such an operator transforms in the $(\mathrm{Sym}(\mathbf{N}^k), \mathrm{Sym}(\overline{\mathbf{N}}^k))$ representation of $\mathrm{U}(N) \times \mathrm{U}(N)$. Thus it can be combined into

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a singlet with C^k. For the case C^{nk} we can consider 't Hooft or monopole operators with n units of flux on each $\mathrm{U}(N)$ factor.[6] Notice that for the particular case of $k = 1$ we can construct in this way chiral primary operators with a single complex field C. These operators have dimension $\Delta = 1/2$ which saturates the unitarity bound, and should thus be free fields. In the dual description, these operators and their complex conjugates describe the center of mass motion of the M2-branes. For $k = 1, 2$ one can construct dimension two currents using 't Hooft operators. By virtue of their dimensions, these are conserved and they enhance the global symmetry to SO(8).[7]

Alternatively, we can note that if we quantize the theory on $S^2 \times \mathbf{R}$ we can diagonalize the oscillator B and go to a highest weight state where only the oscillator corresponding to the first eigenvalue is excited k times. We can then do an analysis similar to what we did on the moduli space and in the $\mathrm{U}(1) \times \mathrm{U}(1)$ case, and conclude that this charge can be canceled by a flux which is only in the first $\mathrm{U}(1)$ factor. We can view the resulting state as a lowest weight state of the representation which is the symmetric product of k fundamentals for the first $\mathrm{U}(N)$ factor and of k anti-fundamentals for the second.

Finally, let us give a more complete description of the chiral primary operators. For this purpose let us concentrate on the chiral primary operators under an $\mathcal{N} = 2$ subalgebra of the full supersymmetry algebra. The chiral primary operators are made with the fields A_i and B_j subject to the relations that follow from the derivatives of the superpotential (2.6). In addition we can construct operators of the form A^k using the 't Hooft (or monopole) operator described above. Of course we can also combine them into operators of the rough form $A^{nk}(AB)^l$, $B^{nk}(AB)^l$. In such operators there are many ways to contract the indices and only some combinations will give protected operators. We can find the protected chiral operators by noticing that these operators are given by considering a certain quantum mechanics on the moduli space, as in [32–34]. The moduli space is $\mathbf{C}^4/\mathbf{Z_k}$. So we have N bosons on this space. More precisely, we imagine that the single particle Hilbert space is that of four harmonic oscillators subject to the constraint that all states have zero $\mathbf{Z_k}$ charge. We then consider N particles on this moduli space. This describes the large N limit of the $\mathcal{N} = 2$ chiral ring. In addition we might have to add some operators of baryonic type, see [35]. We will ignore these extra operators for the time being. The final conclusion is that the "single-trace" operators are given by the one-particle Hilbert space we described above. Once we take into account the full SO(6) symmetry the full space is the same as the space of SO(8) spherical harmonics that are invariant under $\mathbf{Z}_k$. Of course, we also have the full Fock space constructed from products of these.[8]

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[6] The previous version of this paper stated that one could also think of these operators in terms of Wilson lines that end at the insertion point. Though that is correct in the Abelian case, and for Chern-Simons theories with no charged matter [31], it appears to be incorrect in our non-Abelian case when there is charged matter. We thank M. Schnabl for pointing this out to us.

[7] We thank E. Witten for this argument.

[8] This discussion of the spectrum of general chiral primary operators that are charged under the $\mathrm{U}(1)$, which followed [34], is based on the moduli space. Thus, matching the spectrum of chiral primary operators that we found (at large N) to the dual field theory of M2-branes is not really an independent test of the duality. It would be nice to perform a direct operator computation of the chiral ring, in order to display more explicitly the operators in the chiral ring.

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Note that the spectrum of chiral operators becomes SO(8)-invariant for $k = 1, 2$. The $\mathrm{SO}(8)_R$ symmetry mixes the standard operators described at the beginning of this section with the non-standard operators which include 't Hooft disorder operators. This suggests that this symmetry (and, thus, the $\mathcal{N} = 8$ superconformal symmetry) does not act locally on the fields in our Lagrangian. This is related to the fact that the transformation between our description of these theories and a description in which the $\mathcal{N} = 8$ supersymmetry is manifest involves a mirror symmetry transformation in three dimensions (as we will discuss in the next section), which is generally non-local.

In the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theory we have additional baryon-like (or di-baryon) chiral operators of the form $\det(C)$ [36] (with various flavor indices for the C's), carrying N units of $\mathrm{U}(1)_b$ charge. In the $\mathrm{U}(N) \times \mathrm{U}(N)$ theory these operators might not be gauge-invariant, but if we take a product of n of these operators, with nN a multiple of k, we can form a gauge-invariant operator by adding flux as described above. In fact, in this case there is no real distinction between these operators and the gauge-invariant operators described in the previous paragraphs.

Additional interesting operators are Wilson lines. Since we have bifundamentals some Wilson lines can be easily screened by creating excitations of the C fields. If we consider Wilson lines which contain n copies of the fundamental representation of the first $\mathrm{U}(N)$ factor then generically they cannot be screened. However, if we take $n = k$, then the Wilson lines can be screened or become unobservable because they are equivalent to doing a large gauge transformation. More specifically, we can see that if we have a compact space, such as $S^2 \times \mathbf{R}$ and we align n Wilson lines in the fundamental representation of one of the gauge groups along the time direction, then the observable is zero for $n < k$ due to the Gauss law. On the other hand, it can have a non-zero value for $n = k$. In this case we can add a flux on the S^2 and the configuration can become gauge-invariant.

Similarly, if we consider N Wilson lines, they can combine into a singlet under $\mathrm{SU}(N)$. In the $\mathrm{U}(N)$ theory they would still carry charge under the overall $\mathrm{U}(1)$. This charge can be screened by flux if N is a multiple of k, or, if we have nN Wilson lines, when nN is an integer multiple of k.

2.5 The $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theories

The moduli space for the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theories is slightly more complicated than the $\mathrm{U}(N) \times \mathrm{U}(N)$ case described above. The moduli space is still described by diagonalizing all four matrices C_I, but the identifications are more complicated. In the $\mathrm{U}(N) \times \mathrm{U}(N)$ case we had independent $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge transformations acting on the j'the eigenvalue of C_I, C_I^j ($j = 1, \ldots, N$), as $C_I^j \to e^{i(\Lambda^j_{(1)} - \Lambda^j_{(2)})} C_I^j$, and the arguments around (2.8) suggested that we should identify configurations where the Λ's are independently integer multiples of $2\pi/k$ (in addition to the permutation of the eigenvalues). In the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ case the Λ's are constrained by $\sum_j \Lambda^j_{(1)} = \sum_j \Lambda^j_{(2)} = 0$. The effective Chern-Simons term on the moduli space looks like N copies of (2.7), but with the constraint $\sum_j A^j_{(1)} = \sum_j A^j_{(2)} = 0$. If we consider the variation of the Chern-Simons term as in (2.8), we find that the gauge transformations that we need to identify by are the ones for which $k \sum_j \Lambda^j_{(1)} f_j \in 2\pi\mathbf{Z}$ for

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any integers f_j obeying $\sum_j f_j = 0$, since these are the allowed fluxes in the SU(N) theory (and similarly for the $\Lambda_{(2)}$'s). Defining $\omega \equiv \exp(2\pi i/Nk)$, a basis for these identifications is provided by the identifications $g_l : C_I^j \to \omega^{1-N\delta_{jl}} C_I^j$ for $l = 1, \ldots, N-1$, which are all consistent gauge transformations of the type described above.[9] The full moduli space is the space $\mathbf{C}^{\mathbf{4N}}$ divided by the $(N-1)$ g_l identifications and by the permutation group S_N (whose elements do not commute with g_l). It is not clear if there is any simple description of the full group of identifications in this case.[10]

Naively one may think that there is a simple relation between the $\mathrm{U}(N) \times \mathrm{U}(N)$ and the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theories; the former should just arise by gauging the global $\mathrm{U}(1)_b$ symmetry of the latter theory, adding another free U(1) gauge field, and adding appropriate supersymmetric Chern-Simons couplings for the two U(1)'s. This would suggest that the moduli space of the $\mathrm{U}(N) \times \mathrm{U}(N)$ theory should just be an orbifold of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ case, by an argument similar to the one we gave in our analysis of the $\mathrm{U}(1) \times \mathrm{U}(1)$ case. However, this argument is not correct, since U(N) is not simply the product of U(1) and SU(N), but rather it is $(\mathrm{U}(1) \times \mathrm{SU}(N))/\mathbf{Z}_N$, such that the flux quantization conditions are not the same for $\mathrm{U}(N) \times \mathrm{U}(N)$ as for $\mathrm{U}(1) \times \mathrm{SU}(N) \times \mathrm{U}(1) \times \mathrm{SU}(N)$. This is why the moduli space we found for the $\mathrm{U}(N) \times \mathrm{U}(N)$ case is not simply an orbifold of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ moduli space.[11]

2.6 The $N = 2$ case and comparison with the Bagger-Lambert theory

In the special case of $\mathrm{SU}(2) \times \mathrm{SU}(2)$, the bifundamental hypermultiplets are in the real $(\mathbf{2}, \mathbf{2})$ representation, so A_1, A_2, B_1 and B_2 are all in the same representation, and without the superpotential the action (2.2) has an SU(4) flavor symmetry. We prove in appendix A.2 that in this case the superpotential (2.6) also has this SU(4) flavor symmetry. (Note that the 3 + 1-dimensional Klebanov-Witten theory [18] also has an SU(4) global symmetry in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ case [35]).[12] This SU(4) symmetry is a symmetry of the superpotential (written in $\mathcal{N} = 2$ notation) and it acts on the chiral superfields. This should not be confused with the $\mathrm{SU}(4)_R$ symmetry that we had for general N. In addition, we still have the $\mathrm{SU}(2)_R$ symmetry, which exchanges the A's with the complex conjugates of the B's, and does not commute with the SU(4) global symmetry. These two symmetries combine to give an SO(8) global R-symmetry. This SO(8) symmetry rotates all 8 real scalars in the $(\mathbf{2}, \mathbf{2})$ representation. As above, this implies that in this case our theories actually have $\mathcal{N} = 8$ superconformal symmetry (for any value of k), which can be explicitly realized by flavor transformations on the usual supercharges. In fact, one can show that our $\mathrm{SU}(2) \times \mathrm{SU}(2)$

[9] An alternative basis is provided by the elements g_1 and $h_l = g_l g_1^{-1}$ $(l = 2, \ldots, N-1)$, where h_l acts as $C_I^1 \to \exp(2\pi i/k) C_I^1$, $C_I^l \to \exp(-2\pi i/k) C_I^l$.

[10] One can also consider the $(\mathrm{SU}(N) \times \mathrm{SU}(N))/\mathbf{Z}_N$ theory, which has a somewhat different moduli space because of the different fluxes that are allowed.

[11] We could also take different levels for the U(1) and SU(N) factors in U(N), but we will not discuss this possibility here.

[12] In the three dimensional $\mathcal{N} = 4$ super-Yang-Mills theory with this matter content the superpotential breaks this symmetry to an SO(4) global symmetry. However, it turns out that after flowing to the $\mathcal{N} = 3$ Chern-Simons theory, the superpotential (2.6) actually has the full SU(4) global symmetry.

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(or $(\mathrm{SU}(2) \times \mathrm{SU}(2))/\mathbf{Z}_2$) theory is precisely the same as that of [2] (which was written as a Chern-Simons-matter theory in [9]).

For $k = 1$ this theory was claimed to describe two M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$. However, this cannot be precisely correct, since the moduli space does not include the $\mathbf{Z}_2$ transformation acting on each M2-brane separately. This claim was motivated by the Type IIA picture of two D2-branes at an orientifold 2-plane, which lifts to the M-theory $\mathbf{Z}_2$ orbifold. It was assumed that the theory on the D2-branes is the $\mathcal{N} = 8$ SO(4) super Yang-Mills theory, which would then flow to the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ $k = 1$ Chern-Simons theory in the IR. However, the D2-brane gauge group is actually $O(4)$, not SO(4). The theory of two M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$ should therefore correspond to the IR limit of the $\mathcal{N} = 8$ $O(4)$ super Yang-Mills theory. The extra $\mathbf{Z_2}$ in the gauge group provides precisely the extra $\mathbf{Z_2}$ projection in the moduli space. For $k = 2$ the theory was claimed [5, 4] to describe two M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$ with discrete torsion of the 3-form field [37]. This was understood as the low energy limit of the super Yang-Mills theory of two D2-branes at an orientifold 2-plane with discrete RR torsion (the so-called $\widetilde{O2}^-$), which has the gauge group SO(5).

On the other hand, our analysis suggests (and more evidence will follow) that the theory of any number N of M2-branes on the orbifold $\mathbf{R}^8/\mathbf{Z}_k$ is $\mathrm{U}(N) \times \mathrm{U}(N)$ at level k. In particular the theory of two M2-branes on $\mathbf{R}^8/\mathbf{Z}_2$ is $\mathrm{U}(2) \times \mathrm{U}(2)$ at level $k = 2$. The question of discrete torsion is interesting, and we leave it for a future investigation.

3. Brane constructions

Three dimensional gauge theories with a Chern-Simons term can be realized in brane constructions in Type IIB string theory [6, 7]. Generically these theories can have at most $\mathcal{N} = 3$ supersymmetry. In this section we will generalize these constructions to theories with a $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge group, with Chern-Simons terms at levels k and $-k$, respectively, and with matter in the bi-fundamental representation. These theories flow in the IR to precisely the $\mathcal{N} = 6$ superconformal Chern-Simons theories considered in section 2. On the other hand, lifting the configuration (via T-duality) to M-theory will allow us to relate these theories (at low energies) to M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity, and thereby to justify the duality conjecture we made in the previous section.

3.1 Type IIB brane configurations with $\mathcal{N} = 3$ supersymmetry

Our construction of the Type IIB brane configurations will follow closely the approach of [7]. We start with the brane configuration for a three-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory consisting of two parallel NS5-branes along the directions 012345 and separated along the compact direction 6, and N D3-branes along the directions 0126 [38], see figure 1. The directions 012 are common to all the branes, and are identified with the coordinates of our three-dimensional field theories. The D3-branes wind around the compact direction 6, but since they can break on the NS5-branes we get two $\mathrm{U}(N)$ vector multiplets, one for each interval. In addition, the open strings between D3-branes across the NS5-branes give rise to two complex bifundamental hypermultiplets. In $\mathcal{N} = 2$ notation, the bifundamental hypermultiplets give rise to chiral superfields A_i ($i = 1, 2$) in the $(\mathbf{N}, \bar{\mathbf{N}})$ representation, and

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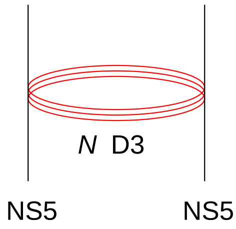

Figure 1: The brane configuration for the $\mathcal{N}=4$ supersymmetric theory in three dimensions with a $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge group and two bifundamental hypermultiplets. We have NS5-branes along 012345 and N D3-branes along 0126.

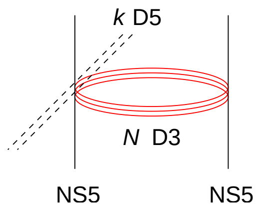

Figure 2: We add D5 branes along 012349 to the configuration in figure 1 so that we get an $\mathcal{N}=2$ theory which is the $\mathcal{N}=4$ theory of figure 1 plus k massless chiral multiplets in the fundamental and k in the anti-fundamental of each gauge group.

B_j $(j=1,2)$ in the $(\bar{\mathbf{N}},\mathbf{N})$ representation.[13] At this point we have an $\mathcal{N}=4$ $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge theory, with a dynamical gauge field, three scalars and four fermions in the adjoint representation of each gauge group, and four complex bosons in the $(\mathbf{N},\bar{\mathbf{N}})$, their complex conjugates in the $(\bar{\mathbf{N}},\mathbf{N})$, and their fermionic partners.

Next, we add k D5-branes along 012349, such that they intersect the D3-branes along 012, as well as one of the NS5-branes along 01234, see figure 2. This breaks the supersymmetry to $\mathcal{N}=2$, and adds k massless chiral multiplets in the fundamental and k massless chiral multiplets in the anti-fundamental representation of each of the $\mathrm{U}(N)$ factors. (Note: this is different from the D5-brane orientation considered in [38], which preserved $\mathcal{N}=4$ supersymmetry.) Each chiral multiplet consists of a two-component Majorana fermion and a complex scalar. We will now recall how to obtain the Chern-Simons term by a mass deformation of this configuration [7]. There are several possible mass deformations. Separating the D5-branes from the D3-branes in the directions 78 corresponds to a standard complex mass parameter in the superpotential, which is inherited from the four-dimensional $\mathcal{N}=1$ theory. The separation in the 5 direction corresponds to a real mass term of equal magnitude but *opposite sign* for the fundamental chiral and anti-fundamental chiral multiplets.

The final deformation, corresponding to a real mass term of *equal sign* for the fundamental and anti-fundamental chiral multiplets, is a *web deformation*, in which the k D5-branes and NS5-branes break along the directions 01234 and merge into an intermedi-

[13] This theory can also be viewed as the dimensional reduction of a four dimensional $\mathcal{N}=2$ gauge theory which arises by a similar construction with D4-branes in Type IIA string theory, or alternatively by considering D3-branes at an $\mathbf{R}^4/\mathbf{Z}_2$ singularity (which is related by T-duality to the D4-brane construction).

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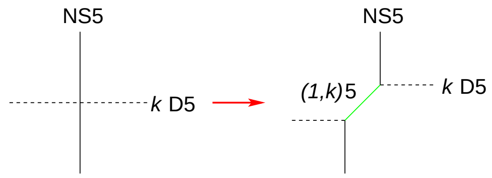

Figure 3: The web deformation of the intersecting NS5-D5 configuration, as seen in the 59 plane.

ate $(1, \pm k)$5-brane, see figures 3, [7]. The sign of the mass term depends on the relative sign of the charges of the intermediate 5-brane. Supersymmetry fixes the angle of the $(1, k)$5-brane (relative to the NS5-brane) in the 59 plane to be [39] (see also appendix B)

$$\theta = \arg(\tau) - \arg(k + \tau) \ , \ \tau = \frac{i}{g_s} + \chi \, . \tag{3.1}$$

In particular for $\chi = 0$ and $g_s = 1$ the angle is $\tan\theta = k$. This is the deformation which interests us. In this case, integrating out the fermions in the chiral and anti-chiral multiplets produces a Chern-Simons term via the parity anomaly [40 − 42]. The coefficient of the CS term gets a contribution of $+1/2$ from each Majorana fermion with a positive mass term, and $-1/2$ from each fermion with a negative mass term. We therefore get a total coefficient k for one of the $\mathrm{U}(N)$ factors, and $(-k)$ for the other. The two CS coefficients have opposite signs because the relative positions of the NS5-brane and $(1, k)$5-brane on the second interval are exchanged relative to the first. Note that if we perform an $\mathrm{SL}(2, \mathbf{Z})$ transformation taking $\chi \to \chi - k$, then this changes the $(1, k)$5-brane into an NS5-brane, and the NS5-brane into a $(1, -k)$5-brane. We would then have on the second interval the same relative positions as on the first interval, except that $k \to -k$.

We end up with an NS5-brane along 012345 and a $(1, k)$5-brane along $01234[5, 9]_\theta$, where $[5, 9]_\theta$ corresponds to the direction $x_5 \cos\theta + x_9 \sin\theta$. This gives at low energies a $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ Yang-Mills-Chern-Simons theory with $\mathcal{N} = 2$ supersymmetry, four massless bi-fundamental matter multiplets and their complex conjugates, and two massless adjoint matter multiplets corresponding to the motion of the D3-branes in the directions 34, which are common to the two 5-branes. The vector multiplet has a mass given by $g_{\mathrm{YM}}^2 k/(4\pi)$. The mass term for the two adjoint fermions in the vector multiplet of each $\mathrm{U}(N)$ has the same sign, and the sign is opposite for the two $\mathrm{U}(N)$'s. Integrating out these fermions at one loop therefore shifts the level of the Chern-Simons term by $\pm N$. However, this shift is canceled by an opposite shift due to the massive gauge field [14].

To get the $\mathcal{N} = 3$ theory we first generalize the construction by rotating the $(1, k)$5-brane relative to the NS5-brane in the 37 and 48 planes. To preserve $\mathcal{N} = 2$ supersymmetry the two angles must be equal. The $(1, k)$5-brane is then along $012[3, 7]_\psi[4, 8]_\psi[5, 9]_\theta$. This gives a superpotential mass proportional to $\tan\psi$ to the two adjoint matter multiplets. For the two fermions in these multiplets the sign of the mass term is opposite, so they do not change the level. For the special case of $\psi = \theta$ the masses of all the adjoint fields are equal, and there is an enhanced $\mathcal{N} = 3$ supersymmetry. In conclusion, the brane configuration under consideration consists of a $(1, 0)$ (or NS) fivebrane along 012345, and

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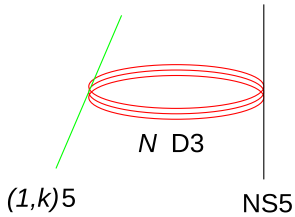

Figure 4: The final brane configuration that gives rise to the $\mathcal{N} = 3$ theory. All branes are stretched along the directions 012. The D3-branes are also stretched along a compact direction 6, and the NS5-branes are stretched also along the directions 345. The $(1, k)$ fivebrane is also stretched along directions mixing the directions 345 and 789, namely $[3, 7]_\theta[4, 8]_\theta[5, 9]_\theta$.

a $(1, k)$ fivebrane along $012[3, 7]_\theta[4, 8]_\theta[5, 9]_\theta$. The angle is given by (3.1). The two branes are separated along the direction 6, and we have also N D3-branes stretched along this compact direction intersecting the two fivebranes, see figure 4. The whole configuration has $\mathcal{N} = 3$ supersymmetry. The $\mathrm{SO}(3)_R$ symmetry corresponds to rotations by the same SO(3) element in the 345 and the 789 subspaces.

In the brane construction it is clear that if we separate m D3-branes from the fivebranes in the transverse directions, so that they do not intersect either of the fivebranes, then at low energies we obtain the $\mathcal{N} = 8$ supersymmetric $2 + 1$ dimensional $\mathrm{U}(m)$ Yang-Mills theory. This brane motion corresponds to giving vacuum expectation values to the bi-fundamentals.

3.2 The lift to M-theory

For simplicity, we will take the axion χ to vanish. We begin by T-dualizing along the direction 6, turning it into the direction $\tilde{6}$ in type IIA string theory. This transforms the D3-branes into D2-branes. The NS5-brane becomes a Kaluza-Klein (KK) monopole along 012345, associated with the circle $\tilde{6}$ (namely, this is the circle which shrinks at the core of the KK monopole). Similarly, the $(1, k)$ fivebrane becomes an object that sits along $012[3, 7]_\theta[4, 8]_\theta[5, 9]_\theta$, which consists of a KK monopole associated with the $\tilde{6}$ circle together with k D6-branes. Of course, these D6-branes are not actual branes, but become flux on the KK monopole [43]. It is again clear that separating the D2-branes from the other objects gives the theory on D2-branes in flat space.

We can now lift the configuration to M-theory, where we get a new direction, 10. The D2-branes become M2-branes, the KK monopole remains a KK monopole associated with the $\tilde{6}$ circle, and the D6-brane becomes a KK monopole associated with the circle in the 10 direction. The $(1, k)$5-brane is therefore lifted to a single KK monopole that is associated with a circle given by a linear combination of $\tilde{6}$ and 10. In other words the two 5-branes in the Type IIB configuration have lifted to a pure geometry in M-theory, which is probed by the M2-branes. The eleven-dimensional geometry is $\mathbf{R}^{1,2} \times X_8$, where X_8 is the space we get by the superposition of the two Kaluza-Klein monopoles. The M2-branes are extended along $\mathbf{R^{1,2}}$ and probe X_8. This eight-dimensional space preserves 3/16 of the supersymmetry [8]. Adding the M2-branes does not break any additional supersymmetry, as long as we add them with the right orientation. Such eight dimensional

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spaces were studied in detail in [8], and we summarize their geometries in appendix B. These spaces are a generalization of the Gibbons-Hawking metric [44], and they obey a linear superposition principle. This allows us to find the explicit geometry for the case at hand. We simply need to add the solutions for each of the Kaluza-Klein monopoles (see appendix B for more details). Since the Kaluza-Klein monopoles have charge one, the M-theory geometry is completely smooth near each of the monopole cores. In fact, near each core it looks like $\mathbf{R^4}$. However, when we look at the intersection region of the two Kaluza-Klein monopoles, we find that the geometry is locally $\mathbf{R^8}/\mathbf{Z_k}$ or $\mathbf{C^4}/\mathbf{Z_k}$. The $\mathbf{Z_k}$ acts as $z_I \to e^{i2\pi/k} z_I$ on the $\mathbf{C^4}$ coordinates. This is explicitly shown in appendix B. This is the only singularity of the metric.

3.3 The IR limit

On the field theory side it is clear that at low energies the $\mathcal{N} = 3$ Yang-Mills-Chern-Simons gauge theory that we constructed flows to the $\mathcal{N} = 6$ superconformal Chern-Simons theory with bi-fundamental fields that we discussed in the previous section. This can be seen explicitly by integrating out all the massive fields. In the M-theory picture, on the other hand, the IR limit becomes the near-horizon limit of the M2-branes probing the $\mathbf{C^4}/\mathbf{Z_k}$ singularity. This singularity preserves 12 supercharges (although the original 8-dimensional space X_8 preserved only 6). The M2-branes do not break any more supersymmetries as long as they have the right orientation. Thus we see that the geometry displays the same supersymmetry enhancement that we found in the field theory discussion of section two. In conclusion, we see that the $\mathcal{N} = 6$ $\mathrm{U}(N) \times \mathrm{U}(N)$ superconformal Chern-Simons theory at levels k and $-k$ that we discussed in section two is the same as the low-energy theory living on N M2-branes probing a $\mathbf{C^4}/\mathbf{Z_k}$ singularity in M-theory, as we conjectured in the previous section.

3.4 The special case of $k = 1$

Note that for the case $k = 1$ we obtain M2-branes in $\mathbf{R}^8$. Thus, the conformal Chern-Simons theories provide a (strongly coupled) field theory description for the theory of M2-branes in flat space. In this description the conformal symmetry is manifest but not all the supersymmetries are manifest; we only see $\mathcal{N} = 6$ supersymmetry, and only an SO(6) subgroup of the R-symmetry is manifest.

In this special case we can also argue that the IR limit of the Yang-Mills-Chern-Simons theory is the same as the IR limit of the $\mathcal{N} = 8$ $\mathrm{U}(N)$ SYM theory, by using the same arguments used for deriving mirror symmetry between the IR limits of $2+1$ dimensional gauge theories realized in type IIB string theory [45, 38], without lifting the configuration to M-theory. Consider the S-dual of the $\mathcal{N} = 3$ brane construction in figure 4. This gives N D3-branes on a circle, intersecting a D5-brane along 012345 and a $(k, 1)$ fivebrane along $012[3,7]_\theta[4,8]_\theta[5,9]_\theta$. In general, the low-energy theory on such branes in rather complicated. However, for $k = 1$ we can shift the axion by 2π and turn the $(1, 1)$ fivebrane into an NS5-brane (without changing its orientation). The low-energy theory on the D3-branes is then a (single) $\mathrm{U}(N)$ $\mathcal{N} = 4$ supersymmetric gauge theory, coupled to an adjoint hypermultiplet and a hypermultiplet in the fundamental representation (from the D3-D5

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strings). This is well-known to be equivalent at low energies to the $\mathcal{N}=8$ SYM theory (without the fundamental hypermultiplet). In the brane language this can be seen from the fact that we obtain the same low-energy theory without the NS5-brane, but then an additional S-duality gives N D3-branes intersecting a single NS5-brane which reduces to the $\mathcal{N}=8$ SYM theory. Since the low energy theories are independent of the axio-dilaton, these arguments imply that the Chern-Simons-matter theory described in the previous section with $k=1$, which arises as the IR limit of our original configuration, has the same IR limit as the $\mathrm{U}(N)$ $\mathcal{N}=8$ SYM theory (which is equivalent to the theory of N M2-branes in flat space).

3.5 $\mathcal{N}=4$ supersymmetric brane configuration

It is interesting to note that there is a brane configuration that is closely connected to our previous discussion which preserves $\mathcal{N}=4$ supersymmetry. In the above discussion we set the Ramond Ramond scalar, χ, to zero. If we consider more generic values of χ, then in order to preserve $\mathcal{N}=3$ supersymmetry we need to rotate the $(1,k)$ fivebrane so as to maintain (3.1). It turns out that for particular values of the τ parameter of type IIB string theory given by

$$\tau=\frac{-k+iak}{1+a^2} \tag{3.2}$$

(for positive a) the angle between the branes becomes $\theta=\pi/2$ so that they are orthogonal to each other. At these particular values the supersymmetry is enhanced from $\mathcal{N}=3$ to $\mathcal{N}=4$. This is shown in more detail using the M-theory lift in appendix B. Changes in τ are not expected to affect the low energy field theory living on the branes. Thus, we see that the IR theory living on this brane is still the conformal Chern-Simons theory that we had above. One can see explicitly (see appendix B) that these changes of the asymptotic moduli do not change the structure of the $\mathbf{R^8}/\mathbf{Z_k}$ singularity that we had in the M-theory lift.

We do seem to have an apparent puzzle, however, since we do not expect to have three dimensional Yang-Mills Chern-Simons theories with $\mathcal{N}=4$ supersymmetry. This puzzle is resolved by noticing that for τ of the form (3.2) we cannot go to weak coupling in the bulk, without doing an S-duality. If we try to produce a small three dimensional gauge coupling on the brane by making the size, L, of the direction 6 very large (recalling that $g_3^2\sim\frac{1}{\mathrm{Im}(\tau)L}$), we find that there are Kaluza-Klein modes with a mass of the order of $1/L$ that we cannot decouple since we cannot make $\mathrm{Im}(\tau)$ large. Thus, we never obtain a weakly coupled $d=3$ $\mathcal{N}=4$ Yang-Mills Chern-Simons theory.

4. The dual gravitational backgrounds of M-theory and type IIA string theory

The discussion of the previous sections suggests that the Chern-Simons-matter theories constructed in section 2 are dual to the conformal field theory living at low energies on N M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. This theory has a dual gravitational description in terms of M-theory on $AdS_4\times S^7/\mathbf{Z}_k$. In this section we discuss various aspects of this gravity dual.

4.1 Supergravity backgrounds

If we write the transverse space to the M2-branes using four complex coordinates z_i ($i = 1,2,3,4$), then the $\mathbf{Z}_k$ quotient we are considering acts as

$$z_i \to e^{i\frac{2\pi}{k}} z_i. \tag{4.1}$$

The gravity dual of N M2-branes in flat space is $AdS_4 \times S^7$, and we simply need to quotient this by this $\mathbf{Z}_k$. Since the $\mathbf{Z}_k$ preserves an $\mathrm{SU}(4)\times\mathrm{U}(1)$ isometry symmetry (acting on the z_i in an obvious way), it is natural to use the description of S^7 as an S^1 fibration over $\mathbf{CP}^3$. In this Hopf fibration the circle has a constant radius, and the $\mathbf{Z}_k$ quotient is simply making this radius smaller [20]. More explicitly, we begin with the $AdS_4 \times S^7$ solution of eleven dimensional supergravity with N' units of four-form flux (we will relate this to N and k later), which has the metric and four-form

$$\begin{aligned} ds^2 &= \frac{R^2}{4} ds^2_{AdS_4} + R^2 ds^2_{S^7}, \\ F_4 &\sim N'\epsilon_4, \\ R/l_p &= (2^5\pi^2 N')^{1/6}, \end{aligned} \tag{4.2}$$

where the metrics $ds^2_{AdS_4}$ and $ds^2_{S^7}$ have unit radius. We can write the metric of S^7 as

$$\begin{aligned} ds^2_{S^7} &= (d\varphi' + \omega)^2 + ds^2_{CP^3}, \\ ds^2_{CP^3} &= \frac{\sum_i dz_i d\bar{z}_i}{\rho^2} - \frac{|\sum_i z_i d\bar{z}_i|^2}{\rho^4}, \qquad \rho^2 \equiv \sum_{i=1}^{4} |z_i|^2, \\ d\varphi' + \omega &\equiv \frac{i}{2\rho^2}(z_i d\bar{z}_i - \bar{z}_i dz_i), \\ d\omega &= J = id\left(\frac{z_i}{\rho}\right) d\left(\frac{\bar{z}_i}{\rho}\right), \end{aligned} \tag{4.3}$$

where φ' is periodic with period 2π and J is proportional to the Kähler form on $\mathbf{CP^3}$. We now perform the $\mathbf{Z_k}$ quotient. For that purpose we write $\varphi' = \varphi/k$, with $\varphi \sim \varphi + 2\pi$. The metric then becomes

$$ds^2_{S^7/\mathbf{Z}_k} = \frac{1}{k^2}(d\varphi + k\omega)^2 + ds^2_{CP^3}. \tag{4.4}$$

Since the volume of this space is smaller by a factor of k than the original volume, in order to have a properly quantized flux on the quotient space we need that $N' = kN$ where N is the final number of flux quanta on the quotient space.

The spectrum of supergravity fields on this background (which are the chiral primaries that are visible in the gravity approximation) is simply the projection of the original spectrum on $AdS_4 \times S^7$ [46] onto the $\mathbf{Z}_k$-invariant states [20, 47].[14] The lowest components of the chiral primary multiplets are [48] in l'th symmetric traceless products ($l = 2,3,\ldots$) of the $\mathbf{8}_v$ representation of SO(8), which decomposes as $\mathbf{4}_1 + \bar{\mathbf{4}}_{-1}$ under $\mathrm{SU}(4)\times\mathrm{U}(1)$. It is easy to see that these precisely match the spectrum of chiral primaries we discussed in

[14] See [20] for a nice table of the U(1)-invariant states.

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section 2, when we identify the U(1) isometry with $\mathrm{U}(1)_b$. We have states that carry no $\mathrm{U}(1)_b$ charge (and are thus preserved by the orbifold projection for any k) and also states that carry a $\mathrm{U}(1)_b$ charge which is a multiple of k.[15]

The radius of the $\mathbf{CP}^3$ factor is large whenever $N' = Nk \gg 1$. However, the radius of the φ circle in Planck units is of the order of $R/kl_p \propto (Nk)^{1/6}/k$. Thus, the M-theory description is valid whenever $k^5 \ll N$, and when k increases the circle becomes small and we can reduce to a weakly coupled type IIA string theory using the usual formulas.

The reduction to type IIA gives the string frame metric, dilaton and Ramond-Ramond forms (setting $\alpha' = 1$) [49, 20]

$$\begin{aligned} ds^2_{\text{string}} &= \frac{R^3}{k}\left(\frac{1}{4}ds^2_{AdS_4} + ds^2_{CP^3}\right), \\ e^{2\phi} &= \frac{R^3}{k^3} \sim \frac{N^{1/2}}{k^{5/2}} = \frac{1}{N^2}\left(\frac{N}{k}\right)^{5/2}, \\ F_4 &= \frac{3}{8}R^3\hat{\epsilon}_4, \\ F_2 &= kd\omega = kJ, \end{aligned} \tag{4.5}$$

where $\hat{\epsilon}_4$ is the epsilon symbol in a unit radius AdS_4 space. We see that we have an $AdS_4 \times \mathbf{CP}^3$ compactification of type IIA string theory (supergravity) with N units of F_4 flux on AdS_4 and k units of F_2 flux on the $\mathbf{CP}^1 \subset \mathbf{CP}^3$ 2-cycle.

The radius of curvature in string units is

$$R^2_{\text{str}} = \frac{R^3}{k} = 2^{5/2}\pi\sqrt{\frac{N}{k}} = 2^{5/2}\pi\sqrt{\lambda}\,, \tag{4.6}$$

where, as before, $\lambda \equiv N/k$ is the 't Hooft coupling. It is interesting that the functional dependence of the curvature on the 't Hooft coupling is the same as in $d = 4$ $\mathcal{N} = 4$ SYM [50]. As expected, for fixed 't Hooft coupling the string coupling goes like $1/N$. Notice that the existence of a weakly coupled string dual was guaranteed by the fact that the field theory had a (discretely) adjustable parameter that enables us to go to weak coupling for fixed N. Therefore one can take the usual 't Hooft limit, which in this case is $k, N \to \infty$ with N/k fixed. Of course, when λ becomes of order one, the approximation of type IIA string theory by supergravity (which we used to write the solution above) breaks down, since the curvature becomes of order the string scale. The field theory is weakly coupled when $\lambda \ll 1$. The type IIA supergravity approximation is valid in the regime where

$$1 \ll \lambda\,, \qquad \frac{N^{1/2}}{k^{5/2}} = \frac{\lambda^{5/2}}{N^2} \ll 1\,, \tag{4.7}$$

while the eleven dimensional supergravity approximation is valid when $N \gg k^5$.

As in other cases of the AdS/CFT correspondence [51], the finite temperature behavior is governed in the gravitational description by an AdS black hole. The finite temperature

[15] In the special case of $k = 1$, the field theory spectrum contains also the state with $l = 1$ describing the center of mass motion of the branes. In the gravity description it is a "singleton" field which can be gauged away to the boundary and is decoupled from the bulk physics.

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partition function (in volume V_2 and temperature T) thus has a behavior which is very similar to the one for M2-branes [52],

$$\beta F = -2^{7/2} 3^{-2} \pi^2 \frac{(Nk)^{3/2}}{k} V_2 T^2 = -2^{7/2} 3^{-2} \pi^2 N^2 \frac{1}{\sqrt{\lambda}} V_2 T^2. \tag{4.8}$$

This expression is valid for $\lambda \gg 1$ (the same expression arises both from eleven dimensional supergravity and from type IIA supergravity), and it receives corrections going (in the large N limit) as inverse powers of $1/\sqrt{\lambda}$. Notice that it has both the characteristic $N^{3/2}$ behavior of M2-branes and the expected N^2 behavior of large N gauge theories for fixed λ. Of course, it would be very nice to derive (4.8) directly from the field theory point of view. At weak coupling ($k \gg N$) the field theory has a free energy given by the free field theory result

$$\beta F \sim -N^2 V_2 T^2 \left[\frac{7\zeta(3)}{\pi} + O(\lambda) \right], \qquad \lambda \ll 1, \tag{4.9}$$

with corrections going as integer powers of λ. We see that as we go to strong 't Hooft coupling the entropy decreases as $1/\sqrt{\lambda}$ giving rise to (4.8). This is different from the behavior of $d = 4$ $\mathcal{N} = 4$ SYM, where the entropy goes to a constant in the strong coupling limit [53].

It is interesting to understand the symmetries of the gravitational solutions. The M-theory solution clearly has an $\mathrm{SU}(4) \times \mathrm{U}(1)$ symmetry. The supercharges are in the $\mathbf{6}_0$ representation of this group [20, 21], so SU(4) is the R-symmetry group and the U(1) is a global symmetry, corresponding to shifts in φ in (4.4). In the type IIA picture the SU(4) symmetry remains as a geometric symmetry of $\mathbf{CP}^3$. We are tempted to identify the U(1) symmetry of M-theory with the RR symmetry in type IIA under which the D0-branes are charged. To a first approximation this is correct. However, we should remember that in an M-theory background with fluxes these symmetries also involve shifts in certain background potentials [46]. In the type IIA picture we might naively think that we have two U(1) gauge fields — the RR 1-form potential, and the 3-form potential integrated over $\mathbf{CP}^1 \subset \mathbf{CP}^3$, whose electric and magnetic charges are carried by D0-branes, D2-branes wrapped on $\mathbf{CP}^1 \subset \mathbf{CP}^3$, D4-branes wrapped on $\mathbf{CP}^2 \subset \mathbf{CP}^3$ and D6-branes wrapped on $\mathbf{CP}^3$. We will see that in the presence of the F_2 flux one combination of these gauge fields is Higgsed and becomes massive. The massless one corresponds to the symmetry generated by

$$J = kQ_0 + NQ_4, \tag{4.10}$$

where Q_0 and Q_4 are the D0-brane and wrapped D4-brane charges, respectively. One way to see this is from the fact that a maximal giant graviton M5-brane [54] (wrapped on $S^5 \subset S^7$) has charge $J = N' = Nk$ in the covering space. After Kaluza-Klein reduction this becomes a D4-brane wrapping the $\mathbf{CP}^2$. We identify the current (4.10) with the baryon number current J_b in the field theory discussion.

From the M-theory point of view it is clear that there is only one massless U(1) gauge field since the only massless gauge fields before doing the orbifold are the SO(8) ones, and the orbifold leaves only one U(1) (and the SU(4), of course). It is instructive to see how

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this comes about from the type IIA point of view. Let us write down the equations of motion involving the RR fluxes and the NS-NS 3-form flux:

$$\begin{aligned}
d\tilde{F}_4 &= -F_2 \wedge H_3\,, \qquad d * \tilde{F}_4 = \tilde{F}_4 \wedge H_3\,, \qquad dH_3 = 0\,, \qquad dF_2 = 0\,,\\
d * (e^{-2\phi} H_3) &= -F_2 \wedge *\tilde{F}_4 + \frac{1}{2}\tilde{F}_4 \wedge \tilde{F}_4\,, \qquad d * F_2 = H \wedge *\tilde{F}_4\,,\\
\tilde{F}_4 &= dA_3 - A_1 \wedge H_3.
\end{aligned} \tag{4.11}$$

Writing down an effective field theory on AdS_4, we naively obtain two U(1) field strengths: F^{D0} containing the AdS_4 components of F_2, and $\tilde{F}^{D2}$ containing the AdS_4 components of the 2-form $\int_{\mathbf{CP}^1} \tilde{F}_4$. We also need to keep the H_3 field in the AdS_4 directions. This will be dual to an axion. Keeping only these fields we then see that the equations (4.11) in the background (4.5) imply the four dimensional equations:

$$\begin{aligned}
&dF^{D0} = 0\,, \qquad *_4\, d *_4 F^{D0} = N *_4 H_3\,, \qquad *_4 d *_4 \tilde{F}^{D2} = 0\,, \qquad d\tilde{F}^{D2} = kH_3\,,\\
&d *_4 (e^{-2\phi} H_3) = NF^{D0} - k *_4 \tilde{F}^{D2}.
\end{aligned} \tag{4.12}$$

We now define $F^{D4} = *_4 \tilde{F}^{D2}$, and the definition of the axion a dual to the B field is deformed to

$$da = *e^{-2\phi} H_3 - NA^{D0} + kA^{D4}. \tag{4.13}$$

Then, the equations for the four dimensional fields become

$$\begin{aligned}
&dF^{D0} = 0\,, && dF^{D4} = 0\,,\\
&*_4 d *_4 F^{D0} = Ne^{2\phi}(da + NA^{D0} - kA^{D4})\,, && *_4 d *_4 F^{D4} = -ke^{2\phi}(da + NA^{D0} - kA^{D4}).
\end{aligned} \tag{4.14}$$

Thus we see that one linear combination of these fields becomes massive (by swallowing the axion field) while the other remains massless. The combination that remains massless can be parameterized in terms of a gauge field A^J as $A^{D0} = kA^J$, $A^{D4} = NA^J$, which is the same as the statement in (4.10).

We should also note that k units of D4-brane charge can be turned into N D0-branes by an NS5-brane instanton. This is easiest to see in the covering space as the statement that $N' = Nk$ units of momentum (N units of D0-brane charge) can become a maximal giant graviton M5-brane [54], which in the quotient space maps to k D4-branes wrapping $\mathbf{CP}^2$.

4.2 Particle-like branes

Let us analyze more explicitly the various particle-like branes in our background. In the M-theory description, $S^7/\mathbf{Z}_k$ does not have any integer-valued homology classes, but it has non-trivial $\mathbf{Z}_k$ homology classes corresponding to the circle and to the product of the circle with $\mathbf{CP}^1$ or $\mathbf{CP}^2$ inside $\mathbf{CP}^3$. We can thus get non-trivial particle-like branes only by wrapping an M5-brane on the circle times $\mathbf{CP}^2$, and non-trivial string-like branes only by wrapping an M2-brane on the circle (leading to the type IIA fundamental string).

In the type IIA description, the D0-brane carries k units of charge under J (it carries k units of momentum on the covering space, since the circle in the orbifold is smaller by

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a factor of k). Thus, it is natural to identify it with the operators in the field theory of the form C^k which were noted in the field theory discussion. The D0-brane is moving in a magnetic field on the $\mathbf{CP}^3$. Thus we expect to have a large number of states, proportional to $\frac{1}{6}\int(\frac{F_2}{2\pi})^3 = \frac{k^3}{6}\int\frac{J^3}{(2\pi)^3} = \frac{k^3}{6}$. This is indeed the scaling in k (at large k) for the dimension of a k'th symmetric product of $\mathbf{4}$'s of SU(4). This gives correctly the chiral primaries with J charge equal to k and lowest energy. In addition we have other states with the same U(1) charge and higher SU(4) representations. In the M-theory description they correspond to spherical harmonics with angular momentum $n > k$ but only k units of U(1) charge in the covering space. These can also be mapped to chiral operators, as discussed in section 2.

The D2-branes wrapped on $\mathbf{CP}^1$ in $\mathbf{CP}^3$ have a worldvolume coupling of the form $\int A \wedge F_2 \sim k\int dtA \wedge J$, where A is the worldvolume U(1) gauge field. This implies that they can only exist if k strings end on them. Thus we see we cannot have an isolated D2-brane, consistent with the fact that this would carry a magnetic charge under a U(1) field that was Higgsed. On the other hand, we can have a D2-brane with k strings ending on it. This is related to the fact that Wilson lines with k fundamentals under one gauge group can be screened, since as usual in AdS/CFT [55] in the 't Hooft limit we can identify the Wilson lines with fundamental strings in the bulk. In the gauge theory this is related to the fact that adding 't Hooft or monopole fluxes can screen the SU(N) quantum numbers. The M-theory lift of a configuration with k strings ending on a D2-brane is simple. As an example we can take k M2-branes that are extended in the time direction and along the z_1 plane. They sit at $z_2 = (\text{const})e^{i2\pi n/k}$ for $n = 0, 1, \ldots, k-1$ and at $z_3 = z_4 = 0$. Then, for large $|z|$ we see that we can neglect the non-zero value of z_2 and we basically have a brane that wraps the U(1) fibration. Since we have k M2-branes the total string charge is k. Near $z_2 \sim$ (const) we should think of the brane as an M2-brane transverse to the circle fibration, which can be interpreted as a D2-brane.

The D4-brane wrapped on $\mathbf{CP}^2$ carries N units of the massless U(1) charge J which we identified with the baryon number charge. From the M-theory point of view this starts its life as a maximal giant graviton M5-brane wrapping an S^5 in S^7. After the $\mathbf{Z_k}$ quotient this is a brane wrapping a non-trivial $\mathbf{Z}_k$ homology cycle that consists of the fiber direction and a $\mathbf{CP^2} \subset \mathbf{CP^3}$. We can identify these wrapped branes with the di-baryon operators discussed in the field theory section.

Finally, the D6-brane wrapped on $\mathbf{CP}^3$ has a coupling on its worldvolume involving $\int A \wedge * F_4$ which implies that N strings should end on it. This is the baryon vertex, as in [56].

Notice that the presence of the D4-brane and the D6-brane suggests that we are not dealing precisely with the U(N) × U(N) theory, since in this theory these objects would only be allowed when N is a multiple of k. Rather, the theory we find resembles the $(\text{SU}(N) \times \text{SU}(N))/\mathbf{Z}_N$ theory, in which this constraint is not present. Presumably, the difference between the U(N) × U(N) theory and the theory that we find lies (as in other examples of the AdS/CFT correspondence) in the behavior of the subtle "singleton" modes that live on the boundary of AdS space [57], since the bulk physics arising from a brane configuration with U(N) gauge symmetry typically captures only the SU(N) dynamics (see, e.g. [58]). It would be nice to elucidate this point further.

Note that the wrapped D2-brane and the wrapped D6-brane both carry a magnetic

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charge under the massless U(1) field (4.10). They also carry magnetic charge under the U(1) that was Higgsed. The latter is responsible for the presence of strings that end on the object. We can form a combination of D6 and D2-branes that carries no charge under the Higgsed U(1). In our conventions, we see from (4.14) that a D6-brane has N strings ending on it whearas a D2-brane has k strings leaving it. Thus if we consider a system of k D6-branes together with N D2-branes then we find that they form an object with no magnetic charge under the Higgsed U(1), so that there need not be any strings ending on it. This looks like a perfectly reasonable localized excitation in the bulk. This excitation, however, carries magnetic charge under the massless U(1) gauge field. The boundary conditions for a U(1) gauge field on AdS_4 only allow for either electric or magnetic charges [59], and in our case the allowed charges are electric, so these branes are not allowed. As explained in [59], one can change the boundary conditions and the dual field theory so that they would be allowed (and the electrically-charged branes would not be allowed).

4.3 High spin operators

A rotating string as in [60] is dual to a high spin operator. We can compute its anomalous dimension in the large λ limit. The result is the same as in [60] when it is expressed in terms of the radius of AdS in string units. In our case we have

$$\frac{R^2_{AdS_4}}{\alpha'} = 2^{\frac{1}{2}}\pi\sqrt{\lambda} = 2^{\frac{1}{2}}\pi\sqrt{\frac{N}{k}}. \tag{4.15}$$

Thus, we find that the anomalous dimension of high spin operators goes like

$$\Delta - S = f(\lambda)\log S, \qquad f(\lambda) = \frac{R^2_{AdS_4}}{\pi\alpha'} = \sqrt{2\lambda}\,, \qquad \lambda \gg 1. \tag{4.16}$$

where $f(\lambda)$ is also the cusp anomalous dimension. As shown in [61], the logarithmic behavior is a general property of any conformal field theory with gauge fields in any number of dimensions. The weak coupling computation was done for general theories in [12][16]

$$f(\lambda) = \lambda^2 + O(\lambda^3)\,, \qquad \lambda \ll 1. \tag{4.17}$$

Of course, it is natural to wonder if this string theory is integrable and whether there is a formula for the cusp anomalous dimension for all λ, as in [62], despite the fact that in our case λ is restricted to rational values.

5. Conclusions

In this paper we constructed and discussed $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons-matter theories with $\mathcal{N} = 6$ supersymmetry. We argued that these theories were equivalent to the low-energy theory on N M2-branes at a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. For $k \gg N$ these theories are

[16] In [12] the general computation was performed for an $\mathcal{N} = 2$ theory with N_f flavors. The leading order term does not depend on the matter interactions. Thus we obtain our result by taking $N_f = 4N$ and multiplying the result in [12] by a factor of two, because in [12] the matter was in the fundamental, where now it is in the bifundamental. In other words, in [12] they considered an open string while here we are considering a closed string.

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weakly coupled, and we argued that for $N \gg 1$ and $k \ll N$ they had dual weakly curved gravitational descriptions in term of M-theory (when $k \ll N^{1/5}$) or type IIA string theory (when $N^{1/5} \ll k \ll N$). The string theory background describes the theory in the 't Hooft large N limit. It would be interesting to understand whether the type IIA string theory is integrable or not.

Since Chern-Simons theories play an important role in condensed matter systems, it is interesting to ask whether there are any other Chern-Simons-matter theories which have a dual gravitational description. The gravity description means that they can be effectively solved at strong coupling. Having such a dual gravitational description predicts many properties of these theories, and might give insight on certain condensed matter systems.

One of the main lessons we can draw is that performing a $\mathbf{Z}_k$ orbifold has allowed us to find an explicit Lagrangian description of interesting M2-brane theories (which becomes weakly coupled for large k). It seems natural to attempt to repeat this procedure in spaces which are not locally $\mathbf{R}^8$ to obtain a variety of theories. Hopefully this can eventually shed some light on the field theory duals of AdS_4 compactifications in the string landscape.

The case $k = 1$ is particularly interesting since our theories are believed to be equivalent to the $\mathcal{N} = 8$ SCFT living on N M2-branes in flat space. We provide an explicit Lagrangian for this SCFT. This Lagrangian displays explicitly only an $\mathcal{N} = 6$ supersymmetry. The spectrum of protected operators in our theory becomes SO(8) invariant for $k = 1, 2$. The SO(8) generators which are not in SO(6) seem to be non-locally realized in terms of the fields in our Lagrangian. In particular, some of the $\mathrm{SO}(8)_R$ generators can transform a field C into $C^\dagger$. Thus, it seems that such generators should involve Wilson lines in the symmetric product of two fundamentals of the first gauge group and two anti-fundamentals of the second. Interestingly, these Wilson lines are unobservable precisely for $k = 1, 2$, which are the cases where we have SO(8) symmetry. So, in these cases the generators may be local operators. It is an interesting problem to construct these generators explicitly. This is analogous to the construction of the supercharges for a compact boson in two dimensions which is at the special radius where it is supersymmetric.

Of course, for $k = 1, 2$ our theory is very strongly coupled, so it is not clear what we can learn about it from our description, except for properties which are protected by supersymmetry. Many of these properties, such as the spectrum of chiral operators, are already known, either from the description of these theories as the IR limit of the $\mathcal{N} = 8$ SU(N) SYM theory or from their description as M-theory on $AdS_4 \times S^7$, and we can verify these results using our description. It is not clear yet how to use our formalism to compute things which are not protected by supersymmetry, such as the entropy of the theory at finite temperature. In principle it might be possible to compute this by putting our theory on a lattice. Of course one could also do such a computation using the alternative definition of these SCFTs as the IR limit of the $\mathcal{N} = 8$ U(N) SYM theory. In our formalism the conformal symmetry is manifest, but not all the supersymmetries are manifest, whereas if we view the theory as an IR limit of $\mathcal{N} = 8$ super Yang-Mills the supersymmetries are manifest but the conformal symmetry is not. For other proposals for a theory of multiple M2-branes in flat space, see [63].

Another interesting problem is the generalization of our discussion to the case of the

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$\mathrm{U}(N) \times \mathrm{U}(M)$ $(\mathrm{SU}(N) \times \mathrm{SU}(M))$ gauge theory. The generalization of our field theory discussion is straightforward, and these theories still have $\mathcal{N} = 6$ superconformal symmetry. In the gravitational dual description, it seems that these theories may be related to turning on $n = N - M$ units of $\mathbf{Z_k}$ torsion G_4 flux in M-theory, if $|N - M| \leq k$. Theories where $|N-M| > k$ do not appear to exist as superconformal theories. Notice that in such theories at least one of the gauge groups would be strongly coupled.

It would also be interesting to understand how to obtain the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theories directly from a string construction. One can consider putting the Klebanov-Witten theory [18] on a compact circle and deforming it by the appropriately fine tuned three dimensional supersymmetric Chern-Simons term. At the level of supergravity this could lead to a flow between a space which is locally $AdS_5 \times T^{1,1}$ and the $AdS_4 \times S^7/\mathbf{Z}_k$ geometry.

It seems that our theories should also admit massive deformations preserving all the supersymmetries, similar to the ones considered in [64].

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A. Explicit verifications of symmetries

A.1 Explicit verification of $\mathrm{SU}(4)_R$ symmetry of the bosonic potential

In this subsection we compute explicitly the bosonic potential and we show that it is invariant under $\mathrm{SU}(4)_R$.

We consider the $\mathrm{U}(N) \times \mathrm{U}(N)$ theory. We have

$$W = \frac{2\pi}{k}\epsilon^{ab}\epsilon^{\dot{a}\dot{b}}\mathrm{Tr}[A_a B_{\dot{a}} A_b B_{\dot{b}}]\,, \quad \frac{\partial W}{\partial A_a} = \frac{4\pi}{k}\epsilon_{ab}\epsilon_{\dot{a}\dot{b}} B_{\dot{a}} A_b B_{\dot{b}}\,, \quad \frac{\partial W}{\partial B_{\dot{a}}} = \frac{4\pi}{k}\epsilon_{ab}\epsilon_{\dot{a}\dot{b}} A_b B_{\dot{b}} A_a\,, \tag{A.1}$$

so that the scalar potential arising from the superpotential is

$$V_{\rm sup} = |\partial W|^2 = \frac{16\pi^2}{k^2}\left(\epsilon_{\dot{a}\dot{b}}\epsilon_{\dot{c}\dot{d}}\mathrm{Tr}[B^\dagger_{\dot{b}} A^\dagger_a B^\dagger_{\dot{a}} B_{\dot{c}} A_a B_{\dot{d}}] + \epsilon_{ab}\epsilon_{cd}\mathrm{Tr}[A^\dagger_b B^\dagger_{\dot{a}} A^\dagger_a A_c B_{\dot{a}} A_d]\right). \tag{A.2}$$

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In addition we find that

$$\frac{k}{2\pi}\sigma_{(1)} = A_a A_a^\dagger - B_{\dot b}^\dagger B_{\dot b}\,,$$
$$-\frac{k}{2\pi}\sigma_{(2)} = B_{\dot a} B_{\dot a}^\dagger - A_a^\dagger A_a \quad \rightarrow \quad \frac{k}{2\pi}\sigma_{(2)} = A_a^\dagger A_a - B_{\dot a} B_{\dot a}^\dagger\,, \tag{A.3}$$

where the Chern-Simons piece of the potential takes the form

$$V_{\rm CS} = {\rm Tr}[A_c A_c^\dagger \sigma_{(1)}^2 - 2A_c^\dagger\sigma_{(1)}A_c\sigma_{(2)} + A_c^\dagger A_c\sigma_{(2)}^2] + {\rm Tr}[B_{\dot c}B_{\dot c}^\dagger\sigma_{(2)}^2 - 2B_{\dot c}^\dagger\sigma_{(2)}B_{\dot c}\sigma_{(1)} + B_{\dot c}^\dagger B_{\dot c}\sigma_{(1)}^2]. \tag{A.4}$$

We would now like to write the sum of these two terms in an $SU(4)_R$-invariant way, using $C_I \equiv (A_1, A_2, B_1^\dagger, B_2^\dagger)$. A bosonic potential which is SU(4) invariant and has the structure of the ones above must be of the form

$$V \sim {\rm Tr}[C_{I_1}C^{\dagger J_1}C_{I_2}C^{\dagger J_2}C_{I_3}C^{\dagger J_3}] \tag{A.5}$$

where the indices need to be contracted in some way. In general, there are four possible contractions (up to cyclic permutations). The most general SU(4)-invariant potential is

$$V = \frac{4\pi^2}{k^2}\big(a_1{\rm Tr}[C_{I_1}C^{\dagger I_1}C_{I_2}C^{\dagger I_2}C_{I_3}C^{\dagger I_3}] + a_2{\rm Tr}[C_{I_1}C^{\dagger I_2}C_{I_2}C^{\dagger I_3}C_{I_3}C^{\dagger I_1}] +$$
$$+a_3{\rm Tr}[C_{I_1}C^{\dagger I_1}C_{I_2}C^{\dagger I_3}C_{I_3}C^{\dagger I_2}] + a_4{\rm Tr}[C_{I_1}C^{\dagger I_3}C_{I_2}C^{\dagger I_1}C_{I_3}C^{\dagger I_2}]\big). \tag{A.6}$$

The coefficients can be determined by matching to the potential we wrote above in simple special cases. We can consider diagonal C's and demand that V vanishes, we can set to zero some of the components of C, etc. In the end we obtain $3a_1 = 3a_2 = -1,\ a_3 = 2,\ 3a_4 = -4$.

In summary, we can write the potential as

$$\begin{aligned} V &= V_{\rm sup} + V_{\rm CS} = \\ &= \frac{4\pi^2}{k^2}\Big(-\frac{1}{3}{\rm Tr}[C_{I_1}C^{\dagger I_1}C_{I_2}C^{\dagger I_2}C_{I_3}C^{\dagger I_3}] - \frac{1}{3}{\rm Tr}[C_{I_1}C^{\dagger I_2}C_{I_2}C^{\dagger I_3}C_{I_3}C^{\dagger I_1}] + \\ &\quad +2{\rm Tr}[C_{I_1}C^{\dagger I_1}C_{I_2}C^{\dagger I_3}C_{I_3}C^{\dagger I_2}] - \frac{4}{3}{\rm Tr}[C_{I_1}C^{\dagger I_3}C_{I_2}C^{\dagger I_1}C_{I_3}C^{\dagger I_2}]\Big) = \\ &= -\frac{8\pi^2}{k^2}{\rm Tr}(C_{I_1}C^{\dagger [I_1}C_{I_2}C^{\dagger I_2}C_{I_3}C^{\dagger I_3]}) + \frac{8\pi^2}{k^2}{\rm Tr}(C_{I_1}C^{\dagger [I_1}C_{I_2}C^{\dagger I_3]}C_{I_3}C^{\dagger I_2}), \end{aligned} \tag{A.7}$$

where the brackets [ ] mean antisymmetrization with a factor of 1/6 or 1/2 respectively. We see that it is indeed invariant under the SU(4) symmetry.

A.2 Enhanced flavor symmetry for gauge group SU(2) × SU(2)

In this appendix we verify that in the case of SU(2) × SU(2) the superpotential (2.6) is invariant under an enhanced SU(4) flavor symmetry rotating the chiral multiplets. In this case we can also think of the fields B_i as bifundamentals, rather than anti-bifundamentals; more precisely we can write

$$(\hat B_i)_\alpha^{\ \beta} = \epsilon_{\alpha\alpha'}\epsilon^{\beta\beta'}(B_i)^{\alpha'}_{\ \beta'} \tag{A.8}$$

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which are bifundamentals. We now would like to write the superpotential in terms of this field.

One can now combine the fields into $E_I = (A_i, \hat{B}_j)$, which are all chiral superfields in the $(\mathbf{2},\mathbf{2})$ representation. We write the SU(4) invariant expression

$$W_{\mathrm{SU}(4)} = \epsilon_{IJKL}(E_I)_{\alpha_1}^{\beta_1}(E_J)_{\alpha_2}^{\beta_2}(E_K)_{\alpha_3}^{\beta_3}(E_L)_{\alpha_4}^{\beta_4}\epsilon^{\alpha_1\alpha_4}\epsilon_{\beta_1\beta_2}\epsilon^{\alpha_2\alpha_3}\epsilon_{\beta_3\beta_4}. \tag{A.9}$$

This is essentially the only way to contract the gauge indices. In fact it is simpler to view the gauge group SU(2) × SU(2) as SO(4) and think of the bifundamentals as carrying an SO(4) index $m = 1, 2, 3, 4$. Then it is clear that the superpotential

$$W_{\mathrm{SU}(4)} = \epsilon_{IJKL}\epsilon_{mnrs}E_I^m E_J^n E_K^r E_L^s \sim \det(E) \tag{A.10}$$

is gauge-invariant and SU(4)-invariant.

It remains to show that this is equivalent to the original superpotential (2.6). This can be shown as follows. The superpotential comes from integrating out the adjoints. In SO(4) notation we can label the adjoint as a self dual tensor φ^+_{mn} and $\tilde{\varphi}^-_{mn}$.

We then have the superpotential couplings

$$\begin{aligned} W &= \frac{k}{4\pi}\varphi^+_{mn}\varphi^+_{mn} - \frac{k}{4\pi}\tilde{\varphi}^-_{mn}\tilde{\varphi}^-_{mn} + \varphi^+_{mn}v_{mn} + \tilde{\varphi}^-_{mn}v_{mn}\,, \\ v_{mn} &\equiv (A_1^m B_1^n + A_2^m B_2^n). \end{aligned} \tag{A.11}$$

After we integrate out φ and $\tilde{\varphi}$ we get (up to a constant)

$$W = P^+_{mn,rs}v_{mn}v_{rs} - P^-_{mn,rs}v_{mn}v_{rs} \sim \epsilon_{mnrs}v_{mn}v_{rs}, \tag{A.12}$$

where $P^\pm_{mn,rs}$ are projectors onto self dual and anti-self dual parts. This then gives

$$W = \epsilon_{mnrs}A_1^m B_1^n A_2^r B_2^s \sim \det(E) = W_{\mathrm{SU}(4)}. \tag{A.13}$$

Thus we conclude that the theory has an additional SU(4) symmetry. Together with the $\mathrm{SU}(2)_R$ symmetry that we had originally ($\mathcal{N} = 3$) this leads to a full SO(8), since the $\mathrm{SU}(2)_R$ mixes the fields E_I^m and their complex conjugates. The SU(4) symmetry of the superpotential should not be confused with the $\mathrm{SU}(4)_R$ symmetry of the bosonic potential that we had in (A.7). In fact, the final bosonic potential in this theory has an $\mathrm{SO}(8)_R$ symmetry, and the action is the same as in [2].

B. Analysis of the M-theory geometry

Let us begin by recalling some facts about Kaluza-Klein monopoles, or Gibbons-Hawking metrics [44]. A Kaluza-Klein monopole is specified by a circle that shrinks at its core, and three spatial dimensions transverse to the monopole. It is of the form [44]

$$\begin{aligned} &ds^2 = U d\vec{x}^2 + U^{-1}(d\varphi + \vec{\omega}\cdot d\vec{x})^2\,, && \varphi \simeq \varphi + 2\pi, \\ &\vec{\nabla}^2 U \equiv \partial_a\partial_a U = 0\,, && \partial_a w^b - \partial_b \omega^a = \epsilon_{abc}\partial_c U, \end{aligned} \tag{B.1}$$

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where both the Laplacian and the epsilon symbol are taken with respect to the flat metric on $\mathbf{R^3}$. The components of $\vec{x}$ have been denoted as x_a, so that the indices a, b, c run over three values. These metrics describe BPS solutions. A particular solution for the function U is

$$U = 1 + \frac{\kappa}{2|\vec{x}|}, \tag{B.2}$$

where κ needs to be an integer in order to avoid singularities. In other words, the quantization condition on the flux of the "gauge field" associated with $\vec{\omega}$ in (B.1) fixes κ to be an integer. When we go to the region $|\vec{x}| \to 0$ we can neglect the 1 in U and set $U = \frac{\kappa}{2|\vec{x}|}$. In this case we get the metric of $\mathbf{R^4}/\mathbf{Z}_\kappa$ or $\mathbf{C^2}/\mathbf{Z}_\kappa$, where the $\mathbf{Z}_\kappa$ acts as $(z_1, z_2) \simeq e^{i\frac{2\pi}{\kappa}}(z_1, z_2)$ on the coordinates of $\mathbf{C}^2$. We can also change the constant 1 in (B.2) to any other constant. This changes the asymptotic size of the circle.

Let us now return to our problem, described in section 3.2. We need to consider eight dimensional spaces X_8 that preserve 3/16 of the supersymmetry. They are called "Toric HyperKähler manifolds", and they were studied in [8]. These metrics involve two circles φ_1, φ_2 (with period 2π) and two sets of 3 spatial directions $\vec{x}^1$, $\vec{x}^2$. The general form of the metric is very similar to the Kaluza-Klein monopole (B.1), except that now the harmonic function U is replaced by a two by two symmetric matrix of functions U_{ij} $(i, j = 1, 2)$ [8]

$$\begin{aligned} ds^2 &= U_{ij} d\vec{x}^i \cdot d\vec{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j)\,, \\ A_i &= d\vec{x}^j \cdot \vec{\omega}_{ji} = dx^j_a \omega^a_{ji}\,, \qquad\qquad \partial_{x^j_a}\omega^b_{ki} - \partial_{x^k_b}\omega^a_{ji} = \epsilon^{abc}\partial_{x^j_c} U_{ki}\,, \end{aligned} \tag{B.3}$$

where U^{ij} is the inverse of the matrix U_{ij}. The matrix U obeys linear equations that follow from this ansatz, see [8]. The metric of a single Kaluza-Klein monopole times $\mathbf{R^3} \times \mathbf{S^1}$ can be written in this form as a configuration with

$$U = U_\infty + \begin{pmatrix} h_1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad h_1 = \frac{1}{2|\vec{x}_1|}, \qquad U_\infty = \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{B.4}$$

The matrix U_∞ encodes the shape of the torus at infinity. We can read off the value of the $\tau = \tau_1 + i\tau_2$ specifying the shape of the torus by writing the asymptotic form of the metric

$$U^{-1}_{ij} d\varphi_i d\varphi_j = (\text{const}) \left[\frac{(d\varphi_2 + \tau_1 d\varphi_1)^2}{\tau_2} + \tau_2 (d\varphi_1)^2 \right] \tag{B.5}$$

or

$$U^{-1}_\infty = (\text{const}) \frac{1}{\tau_2} \begin{pmatrix} \tau_2^2 + \tau_1^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix}, \qquad U_\infty = \frac{1}{(\text{const})} \frac{1}{\tau_2} \begin{pmatrix} 1 & -\tau_1 \\ -\tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}. \tag{B.6}$$

The constant is related to the area of the torus and to the size of the 6th circle in the type IIB picture. Thus, the choice in (B.4) amounts to $\tau_1 = \chi = 0$ and $\tau_2 = 1/g_s = 1$.

Another very simple configuration is given by a diagonal matrix $U = \text{diag}(U_1, U_2)$ with $U_1 = 1 + \frac{1}{2|\vec{x}_1|}$ and $U_2 = 1 + \frac{1}{2|\vec{x}_2|}$. This describes two orthogonal Kaluza-Klein monopoles, each involving a different circle of the two-torus. The solution describing the rotated Kaluza-Klein monopole corresponding to the $(1, k)$ fivebrane has the form

$$U = \mathbf{1} + \begin{pmatrix} h_2 & kh_2 \\ kh_2 & k^2 h_2 \end{pmatrix}, \qquad h_2 = \frac{1}{2|\vec{x}_1 + k\vec{x}_2|}. \tag{B.7}$$

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We will argue below that this metric is non-singular. Since the equations for the matrix U and for ω are linear, we can find solutions by a superposition principle. We thus conclude that the metric corresponding to the type IIB configuration of $(1,0)$ and $(1,k)$ fivebranes at an angle that we described in section 3 is

$$U = \mathbf{1} + \begin{pmatrix} h_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} h_2 & kh_2 \\ kh_2 & k^2 h_2 \end{pmatrix}, \qquad h_2 = \frac{1}{2|\vec{x}_1 + k\vec{x}_2|}, \qquad h_1 = \frac{1}{2|\vec{x}_1|}. \tag{B.8}$$

We can change the asymptotic values of the radii of the circles by changing $\mathbf{1}$ to a more general constant matrix U_∞. Note that the form of the metric and the equations for U are such that they are invariant under $GL(2)$ transformations of coordinates such that the upper and lower indices i,j transform covariantly or contravariantly. (The positions of the i,j indices are important, but the positions of the a,b indices in (B.3) are not.) Such a transformation applied to the above solution would change the asymptotic shape of the φ_i two-torus into a more general one, and it corresponds to the usual $GL(2) = \mathbf{R} \times \mathrm{SL}(2)$ moduli space of supergravity on a T^2. For the time being we will fix the asymptotic values as in (B.8). It is however important to note that if we perform transformations that are in $\mathrm{SL}(2,\mathbf{Z})$ on (B.4), we can get to (B.7) without changing the asymptotic shape of the torus, since the redefinition of the coordinates φ_1, φ_2 is consistent with their periodicities. Since the original metric (B.4) is non-singular, it is clear that (B.7) is also non-singular.

Now let us consider the metric (B.8) which is the superposition of both KK monopoles. We see that near each of the monopoles we reduce to one of the previous solutions (with $\kappa = 1$), and the metric is not singular. So, any singularity could only arise at the point $\vec{x}_1 \sim \vec{x}_2 \sim 0$ where the monopoles intersect. When we are near this point we can neglect the $\mathbf{1}$ contribution in (B.8). We can now do a $GL(2)$ transformation of coordinates of the form

$$\vec{x}'^1 = \vec{x}^1, \qquad \vec{x}'^2 = \vec{x}^1 + k\vec{x}^2. \tag{B.9}$$

With this choice we see that

$$U' = \tfrac{1}{2} \begin{pmatrix} \frac{1}{|\vec{x}'^1|} & 0 \\ 0 & \frac{1}{|\vec{x}'^2|} \end{pmatrix}. \tag{B.10}$$

This U' matrix gives us the superposition of two orthogonal KK monopoles in completely orthogonal directions in the "near horizon limit", where they seem to be simply $\mathbf{R^4} \times \mathbf{R^4}$. However, this is not completely true, because this $GL(2)$ transformation does not preserve the periodicities of the φ_i's. The new φ' coordinates, which are determined by the inverse transformation

$$\varphi'_1 = \varphi_1 - \frac{1}{k}\varphi_2, \qquad \varphi'_2 = \frac{1}{k}\varphi_2, \tag{B.11}$$

do not have the same periodicity conditions as the original ones. The new identifications can be deduced from the original identifications for φ_1 and φ_2. From this we conclude that φ'_1 and φ'_2 have the usual identifications by 2π that would have given a transverse space $\mathbf{R^8}$, plus the extra identification

$$(\varphi'_1, \varphi'_2) \sim (\varphi'_1, \varphi'_2) + 2\pi(-\frac{1}{k}, \frac{1}{k}). \tag{B.12}$$

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We conclude that the KK monopole configuration that is U-dual to the brane configurations that we discussed has a $\mathbf{C^4}/\mathbf{Z_k}$ singularity. In particular, if we define four complex coordinates z_I parameterizing $\mathbf{C^4}$, we see that the identification acts as $z_I \to e^{2\pi i/k} z_I$. In the special case of $k=1$ the manifold is completely non-singular, as it looks like $\mathbf{R^8}$ at the origin.

Note that if we had included the $\mathbf{1}$ in (B.8) then this would have lead to an additional constant contribution to (B.10) of the form $U'_\infty = \begin{pmatrix} 1+\frac{1}{k^2} & -\frac{1}{k^2} \\ -\frac{1}{k^2} & \frac{1}{k^2} \end{pmatrix}$. As explained in [8], a configuration of two orthogonal KK monopoles as in (B.10) plus a constant matrix U'_∞ preserves $\mathcal{N}=3$ (or 3/16) supersymmetry if the matrix U'_∞ is not diagonal, and $\mathcal{N}=4$ (or 1/4) supersymmetry if the matrix is diagonal.

The $\mathcal{N}=4$ brane configuration of subsection 3.5 is very similar except that after the transformation (B.9) the value of U'_∞ is diagonal, of the form $U'_\infty = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}$. As remarked above the configuration is then $\mathcal{N}=4$ supersymmetric (1/4 of the supercharges are preserved). In that case, the full geometry, including the asymptotics, is given by the $\mathbf{Z_k}$ quotient of an orthogonal product of Taub-NUTs, as U' is diagonal. This means that the original configuration is given by (B.8) with $\mathbf{1}$ replaced by U_∞ with

$$U_\infty = \begin{pmatrix} a+\frac{1}{a} & \frac{k}{a} \\ \frac{k}{a} & \frac{k^2}{a} \end{pmatrix}. \tag{B.13}$$

Using (B.6) we see that this corresponds to a τ parameter of the form

$$\tau = \chi + \frac{i}{g_s} = \frac{-k+ika}{1+a^2}. \tag{B.14}$$

Thus, the only difference between the $\mathcal{N}=3$ and the $\mathcal{N}=4$ configurations lies in the asymptotic values of the parameters of the two-torus, or the constant matrix U_∞ that replaces the $\mathbf{1}$ in (B.8). The geometry around $\vec{x}_i \sim 0$ is exactly the same for both configurations. Note that with the matrix (B.13), the asymptotic form of the metric in the two $\mathbf{R^3}$ spaces is not diagonal in the $\vec{x}^i$ coordinates. We can diagonalize the metric by doing again the change of coordinates (B.9), but now only on the coordinates $\vec{x}^i$ and not the angles. This would then give the picture used in subsection 3.5, with a $(1,0)$ brane and a $(1,k)$ brane which are orthogonal in the two $\mathbf{R^3}$ spaces but in a background which has a non-trivial RR axion.

More generally, for an asymptotic metric with U_∞ as in (B.6) we see that the asymptotic metric in the $\mathbf{R^3}\times\mathbf{R^3}$ subspace from (B.3) is not diagonal. We can diagonalize it by choosing new coordinates of the form

$$\tilde{\vec{x}}_1 = \vec{x}_1 - \tau_1 \vec{x}_2\,, \qquad \tilde{\vec{x}}_2 = \tau_2 \vec{x}_2. \tag{B.15}$$

This means that the combinations of coordinates that appear in the harmonic functions h_1 and h_2 corresponding to the $(1,0)$ and $(1,k)$ fivebranes are

$$h_1 \sim \frac{1}{|\tau_2 \tilde{\vec{x}}_1 + \tau_1 \tilde{\vec{x}}_2|}\,, \qquad h_2 \sim \frac{1}{|\tau_2 \tilde{\vec{x}}_1 + (\tau_1+k) \tilde{\vec{x}}_2|}\,, \tag{B.16}$$

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where we neglected an overall constant. This implies that the angle between the $(1,0)$ and the $(1,k)$ fivebranes is given by

$$\theta = \arg(\tau) - \arg(\tau + k). \tag{B.17}$$

For (B.14) we get that $\theta = \pi/2$ so that we have orthogonal branes.

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